Equilibrium Evictions*

Dean Corbae  
University of Wisconsin - Madison and NBER  
Andrew Glover  
Federal Reserve Bank of Kansas City  
Michael Nattinger  
University of Wisconsin - Madison  

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Abstract

We develop a simple equilibrium model of rental markets for housing in which eviction occurs endogenously. Both landlords and renters lack commitment; a landlord evicts a delinquent tenant if they do not expect total future rent payments to cover costs, while tenants cannot commit to paying more rent than they would be able or willing to pay given their outside option of searching for a new rental. Renters who are persistently delinquent are more likely to be evicted and pay more per quality-adjusted unit of housing than renters who are less likely to be delinquent. Evictions are never socially optimal, and lead to lower quality investment in housing and too few vacancies relative to the socially optimal allocation. In our calibrated model, housing externalities widen the gap in housing access and quality between relatively high- and low-earning renters. Finally, government policies that restrict landlords’ ability to evict can improve welfare, though a full moratorium on evictions should be reserved for crises; rent support is generally a better policy than restricting evictions.

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JEL Codes: R28, R30, R31

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1 Introduction

While there is currently a fair amount of empirical work on evictions in economics and sociology (such as the popular book Evicted: Poverty and Profit in the American City by sociologist Matthew Desmond [6]), scant structural work exists on the social costs of eviction. A structural framework can provide a laboratory for conducting policy counterfactuals, such as eviction moratoriums and rental support. To this end, we build a structural model of rental markets in below-median-income neighborhoods and calibrate it to U.S. labor market data for renters, aggregate eviction rates, and landlords’ income from rental units.

We propose a model of directed search in which people with idiosyncratic income fluctuations transition between renting and being unhoused. They are matched with rental units of varying quality owned by landlords who bear the costs of creating vacancies and investing in the quality of their units. An individual’s utility from living in a unit depends on their unit’s quality, but also the overall housing quality in the neighborhood. Such housing quality externalities are discussed in Desmond and Gershenson [7] and empirically supported by Autor, et al [2] and Diamond and McQuade [9]. We use the model to measure the positive and normative response of evictions and vacancy creation in the rental market both in the steady state and in response to aggregate shocks that increase unemployment, such as the Covid-19 crisis.

In our model, the search process is directed to a particular submarket (as in Moen [18] and Menzio and Shi [17]) rather than random (as in Mortensen and Pissarides [19]). On one side of the market, landlords choose the quality of the rental and offer a menu of rental contracts when creating vacancies. On the other side of the market, renters choose what type of housing vacancies to apply to and pay rent as long as they are employed, but face heterogeneous risks of unemployment spells, during which they cannot afford to pay rent. Renters and landlords searching for each other are brought into contact by a constant-returns-to-scale matching function, with search on both sides directed to submarkets. Each submarket is defined by the quality of the rental unit, the monthly rent that the renter agrees to pay, and renter characteristics or “type” (here proxied by their employment prospects). Importantly, the competitive search equilibrium is conditionally block recursive: the renter and landlord’s values of searching in a given submarket only depend on the equilibrium distribution of housing and employment through the externality. Conditional block recursivity allows us to easily study aggregate shocks to the labor market, such as the Covid-19 pandemic.

The critical friction for evictions to occur in equilibrium is two-sided lack of commitment. Barring legal constraints, a landlord who is not being paid rent can evict their tenant and search for a paying one, while a renter has no commitment to remain in a unit if they would
be better off looking for a new rental. The landlord of an unemployed tenant incurs costs without receiving rent and therefore requires an increase in future rent in order to allow the tenant to remain. However, once the tenant is re-employed there is a limit on how much they are willing to pay since they can search for a new rental instead. Eviction occurs whenever the landlord requires more to keep an unemployed tenant than what the tenant would be willing to pay upon re-employment. In this case, continuing with the match would deliver a negative surplus to the landlord, whereas their outside option of posting a new vacancy delivers zero, so they evict the tenant. In contrast, a benevolent social planner would never destroy a match once it has been created since the social surplus must have been positive in the first place; how the surplus is split between landlord and renter is irrelevant to the planner.

We calibrate the model to match salient features of rental and labor markets including the share of renters who are evicted each year, the income-gradient of rent burdens, the quality gradient of rent-to-quality ratios, and the spillover from an increase in the quality of some rentals in a neighborhood to the value of nearby units. Importantly, our model gives an equilibrium perspective on what Desmond and Wilmers [8] call “exploitation”. They find that low-quality rentals have much higher rent relative to their market value than do high-quality rentals. However, in our model expected-discounted profits are identically equal to zero across all units ex-ante, but this leads to higher rent relative to quality in low-quality units because landlords know that lower profits will accrue ex-post from units where eviction is likely. They respond by investing less in such units while charging relatively high rent in order to generate revenue before eviction occurs. Similarly, our equilibrium allows us to map the estimates of housing externalities from Autor, et al [2] into housing and welfare inequality. We find that housing externalities raise the welfare of higher-income renters by 3.3 percent more than lower-income renters.

In the calibrated equilibrium, evictions occur and are suboptimal. As a consequence, the competitive equilibrium features much lower housing supply for people who face eviction risk — at the extreme, there is no supply of housing for these individuals when they are unhoused and unemployed, whereas the planner would only let them to be unhoused for 53 days on average. Furthermore, the housing that landlords to supply for high-eviction risk tenants is only 61 percent as good as the planner would like. Less supply and lower quality investment by landlords leads to lower aggregate housing quality, which spills over to renters who are not at risk of eviction as well. On net, aggregate welfare from the competitive equilibrium is 18.3 percent lower than from the planner’s allocation (three-quarters of which is due to the commitment friction and one quarter due to the neighborhood externality).

Given that evictions lead the competitive equilibrium to be suboptimal, we use the model
to evaluate policies that reduce evictions. The most direct policy is what we call "eviction restrictions", which reduce the probability that a landlord who would like to evict their tenant are allowed to do so. We find that some restrictions are optimal in order to reduce the number of positive surplus matches destroyed ex-post. However, severe restrictions reduce landlord profits, which leads them to supply less housing and reduce quality investment ex-ante (along the lines of the unintended consequences of firing costs that reduce labor demand in Hopenhayn and Rogerson [11]). We find that the optimal eviction policy forces landlords to wait just over two months, on average, before being allowed to evict a delinquent tenant. In contrast to eviction restrictions, rent support paid to landlords with unemployed tenants can eliminate evictions and actually increase housing supply. With externalities, rent support may be Pareto improving because it increases overall housing quality, which raises the welfare of renters who pay taxes to finance rent support but never face eviction risk themselves. Finally, we leverage the model’s conditional block recursivity to show that a full eviction moratorium can raise welfare if temporarily imposed during a deep crisis in which separation rates rise and job-finding rates fall dramatically (such as the Covid-19 pandemic).

Literature

The only other structural papers on rental evictions that we are aware of are Abramson [1] and Imrohoroglu and Zhao [13]. These papers focus more on the details of the demand side of rental housing, whereas our main contributions are on the supply side. First, they focus on the decision of renters to go delinquent, but treat the landlord’s eviction choice as exogenous; we endogenize the landlord’s decision to evict a delinquent tenant and the resulting inequality in housing outcomes. Second, they do not model search and matching frictions in rental markets, but assume that an unhoused person finds a new rental as soon as they can afford the rent; in our model, a person may be persistently unhoused even after finding a new job. Our competitive search framework also allows us to characterize rental market tightness. Third, they assume that housing sizes are exogenously given and indivisible, which leads to some people being homeless because they cannot afford the lowest-sized housing; we endogenize housing quality and match data on heterogeneity in rent-to-quality across units. Finally, they do not characterize the socially efficient allocation of housing as a point of comparison for their competitive equilibria and policy counterfactuals; we use the socially optimal allocation to isolate the market failures that lead to inefficiency in competitive equilibria.

Specifically, Abramson builds an overlapping generations model of households who face idiosyncratic income and divorce risk. Households rent houses from real-estate investors by signing long-term noncontingent leases specifying a per-period rent which is fixed for the duration of the lease. Since contracts are non-contingent, households may endogenously
default on rent (and do so in equilibrium). An eviction case is filed against a default. Each period the household is in default, it is evicted with an exogenous probability that captures the strength of tenant protections against evictions in the city. Once evicted, an unhoused person can move into another rental as soon they can afford the rent and prefer doing so to remaining unhoused. Similarly, Imrohoroglu and Zhao build a consumption-savings model in which households face income and health shocks and choose whether to be home owners or renters, as well as the type of house they live in and whether or not to pay their rent, with eviction taken as an exogenous outcome based on the renter’s decisions.

Organization

The paper is organized as follows. Section 2 describes data facts. Section 3 lays out the model environment. Section 4 solves for the efficient level of rental quality and tightness. Section 5 describes a decentralized competitive search equilibrium with fixed rental rate contracts and discusses the implications of lack of commitment. Section 6 calibrates a steady-state version of the model and describes its properties relative to the planner’s allocation. Section 7 analyzes the welfare effects of both eviction restrictions and direct rental subsidies in a stationary equilibrium. Finally, Section 8 considers optimal policies during crisis events and Section 9 concludes.

2 Empirical Facts

We use a combination of empirical facts and our own data analysis to motivate and discipline our structural model. We first list these facts and then present analysis from the Survey of Consumer Finance (SCF), the Current Population Survey (CPS), and the Rental Housing Finance Survey (RHFS).

- About 35 percent of U.S. households rent rather than own their homes (CPS).

- In a typical year, 2 – 3 percent of renting households are evicted (Eviction Lab).

- Eviction is more likely among low-income renters. Collinson, et al. [5] find that people who have an eviction filed against them earn only $300 per week, on average, during the two years preceding eviction.

- Renters are twice as likely to be evicted after losing their jobs (Desmond and Gershenson [7]).

\[1\] Appendix A provides details of variable definitions and further discussion of sample selection used in our statistics from the SCF, CPS, RHFS.
- Renters have low net worth: about $6300 for the median renter in 2019 (SCF). Of this, the median renter had only $1100 in cash-like assets (checking and savings accounts) and a quarter of renters had under $120. The median rent was $830. Lower-income renters have almost no liquid assets.

- Among renters, we estimate that 43 percent are hand-to-mouth (based on the definition from Kaplan, Violante, and Weidner [14]) and 57 percent would be unable to cover rent plus half of their typical bi-weekly income. For renters below median income, 72 percent are hand-to-mouth.

- Rent as a share of income (the rent burden) is declining in renter income, ranging from 30 to 50 percent for households below median income (SCF). ²

- We calculate that rent is lower, relative to market value, for units with high valuations (RHFS). Similarly, Desmond and Wilmers ([8]) find that rent is more similar between poor and nonpoor neighborhoods than property values which are substantially higher in nonpoor neighborhoods.

- Autor, Palmer and Pathak ([2]) estimate that changes in rental unit market value spill over to similar units. After Cambridge, MA eliminated rent control, the market value of rent controlled units rose about twice as much as similar non-controlled units, but non-controlled units (which were not directly affected by the end of rent control) still saw price appreciation.

### 2.1 Low-Income Renters are Hand-to-Mouth

We use the 2019 Survey of Consumer Finance to decompose the median renter’s financial net worth into liquid assets (checking and savings accounts), illiquid assets, and debt. We define a renter as someone who reports a positive monthly rent for housing services and restrict our sample to households between the ages of 25 and 70. We also use the definition of hand-to-mouth from Kaplan, Violante, and Weidner [14] (i.e. liquid wealth less than half of biweekly income) to estimate that the share of renters who are hand-to-mouth is 43 percent overall and 72 percent if we include rent commitments and look at lower-income renters in the SCF (i.e. those below median income). Table 1 reports the median and bottom quartile value for rent, liquid assets, and income for all renters, but also for lower-income renters.

²Abramson [1] finds slightly higher numbers for specific cities.
Table 1: Summary Statistics for Renters in SCF

<table>
<thead>
<tr>
<th>Variable</th>
<th>Overall Median</th>
<th>25th Pctile</th>
<th>Low Income Median</th>
<th>25th Pctile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rent</td>
<td>$860</td>
<td>$600</td>
<td>$690</td>
<td>$500</td>
</tr>
<tr>
<td>Liquid Assets</td>
<td>$1020</td>
<td>$100</td>
<td>$250</td>
<td>$0</td>
</tr>
<tr>
<td>Networth</td>
<td>$6700</td>
<td>$10</td>
<td>$2590</td>
<td>$0</td>
</tr>
<tr>
<td>Income</td>
<td>$38,688</td>
<td>$21,380</td>
<td>$21,380</td>
<td>$14,254</td>
</tr>
</tbody>
</table>

Overall the median rent in 2019 was $860, which was 43 percent higher than the bottom quartile $600. On the other hand, median liquid assets was over ten times the bottom quartile and median income was nearly twice the bottom quartile, which means that the rent burden is falling with income. In fact, the average rent burden falls from 48 percent to just 27 percent when income rises from the bottom to the second quartile of household income and continues to fall with income, as can be seen in Figure 1 where we plot the average rent burden against the average income within each income quartile.

Figure 1: Rent Burden by Income, 2019 SCF

This data suggests that, for the population of low-income renters for whom our model is most appropriate, it is unlikely that missed rent could feasibly be capitalized into future payments. Furthermore, an unemployed renter would be unable to pay rent out of liquid savings and has little wealth even including illiquid assets.
2.2 Renter Employment and Earnings Dynamics

Renters in our model differ by income when employed, but also in the probabilities of finding or keeping a job. We use the panel dimension of the Current Population Survey to estimate transition matrices and relative incomes for renters with high and low employment propensities. We use data from 2018 to 2019 and restrict the sample to include only those individuals aged 25 to 70 who reported renting their housing for at least one month, were in the labor force for at least one month (out of a possible eight), and had reported positive average earnings that were below the median. Within that group, we characterize type L individuals as those who were in the bottom decile of employment rates, which corresponds to those who were employed less than half of their interview months. After defining our types in this way, we estimate the following earnings and job finding/keeping rates:

Table 2: Labor Market Outcomes for Low-Income Renters in CPS, 2019

<table>
<thead>
<tr>
<th></th>
<th>High Employment</th>
<th>Low Employment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly Earnings When Employed</td>
<td>$1025</td>
<td>$501</td>
</tr>
<tr>
<td>Job Finding Rate</td>
<td>0.89</td>
<td>0.17</td>
</tr>
<tr>
<td>Job Separation Rate</td>
<td>0.04</td>
<td>0.43</td>
</tr>
<tr>
<td>Fraction of each type</td>
<td>0.9</td>
<td>0.1</td>
</tr>
</tbody>
</table>

From this data, we use the ratio of type L earnings to type−H earnings (we normalize the type L’s level in the model) as well as the finding and keeping rates to discipline the Markov Chain on employment status for each type in Table 2. By definition, then, the share of L−type households in the population we focus on in our model is 10 percent and the share of H−type is 90 percent. Table 2 documents that type H finding rates are over four times that of type L and type H retention rates are almost double that of type L.

We acknowledge that there is a fundamental challenge to estimating employment and income processes for people who are likely to be evicted due to attrition. The CPS interviews members from a given address from month to month, which means that somebody who is evicted will not be in the same housing unit for a follow up interview. Therefore, we will miss people who report being unemployed and move before being interviewed again. This attrition likely biases our job-finding rates upward, since we are oversampling those with relatively short unemployment durations who find a job quickly enough to avoid eviction before their next interview. While over-estimating the job-finding rate of individuals at risk for eviction could affect our precise quantitative results, a lower job-finding rate for type L individuals would only strengthen the incentive for landlords to evict them. We are less concerned about bias in the separation rate, since somebody who is interviewed the month
before losing their job is likely to remain in the same unit the following month as well.

### 2.3 Rental Value, Rent Rates, and Landlord Profits

Our final data source is the 2018 Rental Housing Finance Survey, which is a cross-sectional survey of landlords that asks detailed information about their rental units, their income from the rentals, their costs, and the market value of their units. We use this data to measure the fixed cost of each unit for the bottom two quartiles of rental units by market value, as well as the ratio of rent-to-market value between the first and second quartile. Table 3 provides these measures in 2018 dollars.\(^3\)

<table>
<thead>
<tr>
<th>Market Value Pctile</th>
<th>Rent</th>
<th>Operating Costs</th>
<th>Market Value</th>
<th>Flow Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom 25</td>
<td>$474</td>
<td>$173</td>
<td>$21,424</td>
<td>$301</td>
</tr>
<tr>
<td>26 — 50</td>
<td>$643</td>
<td>$294</td>
<td>$63,993</td>
<td>$349</td>
</tr>
</tbody>
</table>

We will match our model’s predicted ratio of rent-to-quality for high quality relative to low quality units to the empirical rent-to-market-value ratios of high to low value units. We will also discipline the cost of keeping a unit occupied using the operating costs from the RHFS.

### 2.4 Spillovers

Many studies find evidence that housing markets exhibit positive neighborhood externalities. For example, Diamond and McQuade [9] find that housing built for low-income residents spills over to reduce home prices in high-income neighborhoods. Autor, Palmer and Pathak [2] provide estimates that are most easily mapped into our structural model’s outcomes. They look at the market value of units in Cambridge, MA as rent control was lifted. Importantly, Cambridge had both controlled and uncontrolled rentals in close proximity to one another, in neighborhoods with many rent controlled units and in neighborhoods with few rent controlled units.

Table 4 shows their estimates and standard errors. POST is an indicator that takes the value 0 before the law change eliminating rent control and 1 after. RCI is a continuous index for the share of previously rent-controlled housing in the 0.2-mile radius around the unit. RC and NON-RC are indicators for whether a specific unit was originally subject to rent

\(^3\)We describe the calculation of operating costs in Appendix A.
control (RC = 1) or not (NON-RC=0). The point estimates suggest that the market value of rent controlled units rose by 25 log points, but even units that were not previously subject to rent control saw an increase of 13 log points. We therefore discipline spillovers in our model so that a change in rent that increases affected rentals leads their value to increase by twice as much as unaffected rentals. However, since their estimate for the spillover is not significant at the 5 percent level, we will also consider an economy without spillovers.

Table 4: Spillover Estimates from Autor, Palmer, and Pathak

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>POST × RCI × RC</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
</tr>
<tr>
<td>POST × RCI × NON-RC</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
</tr>
</tbody>
</table>

We will use these estimates to discipline the spillover of aggregate rental quality onto the flow utility of housing for an individual renter. Specifically, we will conduct an experiment where randomly assign some units to have rent control and choose the parameter governing housing externalities to match the relative change in quality for non-controlled units to that of controlled ones.

3 Environment

There is a unit measure of people of two types $i \in \{H, L\}$ who live for an infinite number of discrete periods. The fraction of type $i$ is denoted $\mu_i$ which we will take from data in Table 2. People can be either housed ($j = h$) or unhoused ($j = u$) and either employed ($e = 1$) or unemployed ($e = 0$), meaning they can be in one of four states at any point in time.

The two types of people differ in the probability of being employed in the next period given by a type dependent Markov Process $p_{i,e,e'} = Pr(e' | i, e)$ where $(e', e) \in \{0, 1\} \times \{0, 1\}$. They also differ in their income from employment $y_{i,e=1} = y_i$. We assume that $H$–types are more likely to keep or find a job, i.e. $p_{H,e,1} > p_{L,e,1}$ for all $e$. Further, conditional on being employed, type $H$ have higher income $y_H > y_L > \alpha$. Thus, type $H$ have a higher job finding rate, a lower separation rate, and higher expected lifetime earnings than type $L$ consistent with the data in Table 2. An unemployed household generates $y_{i,e=0} = \alpha$ units of the consumption good.

People have linear utility over housing $U^j$ for $j \in \{u, h\}$ and their consumption of non-housing goods $C$ above a subsistence threshold $\alpha$. That is, flow utility is given by $C - \alpha + U^j$
with $C \geq \alpha$. Housed utility depends on both the quality of one’s own housing ($q$) as well as the total quality of all housing ($Q$), which we interpret as a positive spillover externality. Specifically, the period utility for a given person of type $i$ living in housing of quality $q$ is $U^h = q \cdot \mathcal{E}(Q)$, with $\mathcal{E}(0) = 1$ and $\mathcal{E}'(Q) \geq 0$. Our interpretation of the externality ($\mathcal{E}(Q)$) is that people like to be surrounded by high-quality housing in their neighborhood, so the externality operates through the quality of neighboring units, not the income or employment of the residents of those units. We use estimates from Table 4 to discipline the parameterization of this externality. We normalize the flow utility of an unhoused person $U^u = 0$. People discount utility across periods with factor $\beta$.

Matching unhoused people to new housing takes time due to search frictions. Specifically, if there are $V$ vacant housing units and $U$ unhoused people in period $t$, then $M(U, V)$ new matches between houses and unhoused people will be created for $t + 1$. We assume that $M$ has constant returns to scale and define tightness as $\theta = \frac{U}{V}$, the rental finding rate as $\phi(\theta) = \frac{M(U, V)}{U} = M(1, \theta^{-1})$ with $\phi'(\theta) < 0$, and the rental filling rate as $\psi(\theta) = \frac{M(U, V)}{V} = M(\theta, 1)$ and $\psi'(\theta) > 0$. Hence it is hard (easy) to find (fill) a rental unit in a tight market. A housed person separates from her housing unit with exogenous probability $\sigma$ in each period. Once a separation occurs, the unit’s quality depreciates fully.

Creating a new housing unit costs $\kappa$ units of utility up front and having an occupant of type $i$ in the unit costs $f_i$ units each period with $f_H \geq f_L$. We use estimates of $f_i$ consistent with the data in Table 3. Furthermore, the unit’s quality, $q$, requires a one-time investment $c(q - f_i)$ units of utility after the match occurs. In order to ensure that positive surplus matches can be created, we assume that $c'(q - f_i) \geq 0, c''(q - f_i) \geq 0, c(0) = 0$, and $c'(0) = 0$. That is, it is costless to put $q = f_i$ and even costless to increase $q$ marginally above $f_i$, at which point the social surplus of having a person in the housing unit is strictly positive.

The timing in any given period is as follows:

1. New housing is created at cost $\kappa$.

2. People receive income $y_i$ if employed and income $\alpha$ if unemployed.

3. Housed people receive utility $q \cdot \mathcal{E}(Q)$ from housing services while unhoused people receive zero utility.

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4Since we have a unit measure of people and the quality of unhoused is zero, $Q$ is also the average quality.

5We note that the housing definition of “tightness” is opposite that of its definition in labor search.

6For simplicity, we assume that enough physical rental units are available that free entry holds. The costs of investing in quality are thought of as refurbishment of the unit for new residents (e.g. repainting, fixing or replacing appliances, etc.).

7We allow for $i$—dependence of the fixed costs in order to capture the empirical fact that higher quality units incur higher operating costs. In our model, high employment propensity renters have high quality housing, so we make the fixed costs type dependent for simplicity.
4. Unhoused people match with housing according to $M(U, V)$.

5. Newly matched housing units receive quality investment $q$ at cost $c(q - f_i)$.

6. Housed people become unhoused with probability $\sigma$.

7. Employment status changes according to Markov process described above, independent from housing status.

4 Socially Optimal Housing

Consider a social planner who chooses housing outcomes $(q_{i,e}, \theta_{i,e})$ for $(i, e) \in \{H, L\} \times \{0, 1\}$ to maximize the discounted utility of households. At any date, the planner begins in a state with total housing quality $Q$ and measures of the population $\mu_{i,e}^u$.

We study a social planner’s problem in which evictions never occur. This is logically true because the social surplus $q - f_i$ of a match is constant over time, therefore if the planner ever chose to create a match with quality $q$, then they would never optimally destroy it. Further, our assumption that additional quality is costless at $q = f_i$ implies that the planner chooses to create matches (all of which have positive surplus). This implies that every unhoused person will have a positive probability of being matched and that the law of motion for $Q$ is given by

$$Q' = (1 - \sigma)Q + \sum_{i \in \{H, L\}} \sum_{e \in \{0, 1\}} \mu_{i,e}^u \phi'(\theta_{i,e}) q_{i,e}. \text{8}$$

The socially optimal (or first-best) stationary allocation solves

$$c'(q_{i,e} - f_i) = \beta \frac{\mathcal{E}(Q) + Q \mathcal{E}'(Q)}{1 - \beta(1 - \sigma)} \quad (1)$$

$$\kappa - \theta_{i,e}^2 \phi'(\theta_{i,e}) c(q_{i,e} - f_i) = \beta \theta_{i,e}^2 \phi'(\theta_{i,e}) \left[ \frac{\mathcal{E}(Q) + Q \mathcal{E}'(Q) - f_i + \theta_{i,e}^{-1}(\kappa + c(q_{i,e} - f_i) \psi(\theta_{i,e}))}{1 - \beta(1 - \sigma - \phi(\theta_{i,e}))} \right] \quad (2)$$

$$Q = \frac{1}{\sigma} \sum_{i \in \{H, L\}} \sum_{e \in \{0, 1\}} \mu_{i,e}^u \phi(\theta_{i,e}) q_{i,e} \quad (3)$$

The socially optimal quality choice in (1) sets the marginal cost of providing housing quality to its expected marginal benefit. The socially optimal choice of rental tightness in (2) sets the marginal cost of posting a vacancy to the expected marginal increase in social surplus. Finally, equation (3) determines the neighborhood externality in a stationary allocation.

Notice that equations (1) through (3) do not depend on employment status $e$ explicitly (i.e. the only place $e$ enters (1) and (2) is in $q_{i,e}$ and $\theta_{i,e}$ and not in the functions themselves.

8The entire planner’s problem is described in Appendix F.
while $e$ is integrated out in (3)). We say that the social planner is conditionally egalitarian — they choose quality and tightness to be type dependent but not employment dependent (i.e. $q_i$ and $\theta_i$). We will now show that a realistic decentralized equilibrium is inefficient, in general, for three reasons. First, landlords may choose to evict tenants who lose their jobs and are expected to have long unemployment spells. Second, employed rental seekers will have a higher probability of finding a house (i.e. $\phi(\theta_{i,1}) > \phi(\theta_{i,0})$) because landlords will post fewer vacancies for the unemployed than the employed because they will not be able to accrue as many months of rent on average. Third, landlords will not fully internalize the externality from creating a new match or investing more quality in a given match, so that total quality in the decentralized economy will be lower than what a social planner would choose.

5 Decentralized Equilibrium

Our baseline decentralized equilibrium is designed to match the features of rental markets for lower-income people. Landlords post vacancies and invest in the quality of the rental units they create. We assume that renters must pay rent whenever employed and that a landlord cannot evict somebody in a period when rent is paid but can choose to do so after the person misses rent. Unhoused people direct their search to rentals of quality $q_{i,e}$ in submarkets of tightness $\theta_{i,e}$. The fixed terms of the rental contract $r_{i,e}$ must compensate the landlord for the vacancy creation, quality creation, and upkeep costs ($\kappa, c(q - f_i), f_i$).

Empirically, Section 2 showed that lower-income renters (with incomes below the median income of all renters) have few liquid assets, especially relative to rent. Specifically, median liquid assets for that group are $250 while median rent is $690, for the bottom quartile rent is $500, and both liquid assets net worth are zero. Therefore an unemployed renter would be unlikely to have enough money to pay rent and renters would have a hard time paying missed rent once they found another job. For this reason, we focus our model of low income renters as hand-to-mouth lacking savings or the ability to pay missed past rent.

A landlord who has a renter in state $(i,e)$ with constant rent $r$ and housing quality $q$ has

\begin{footnotesize}

9Furthermore, equations (1) through (2) only depend on type through differences in fixed costs, $f_i$, not through differences in employment rates. Therefore, if fixed costs were the same across types then the social planner’s allocation would be fully egalitarian — quality and tightness would be independent of both employment status and type $i$.

10Landlords would not choose to evict an employed renter even if they were allowed to do so, since the discounted expected profits must have been positive in order for the landlord to have posted the vacancy in the first place. See Appendix C for a proof that $\epsilon_{i,1}(r, q) = 0$.

\end{footnotesize}
the following values:

\[ L_{i,1}(r, q) = r - f_i + \beta (1 - \sigma) \sum_{e' \in \{0, 1\}} p_{i,1,e'} L_{i,e'}(r, q), \]  

\[ L_{i,0}(r, q) = \max_{\epsilon \in \{0, 1\}} -f_i + \beta (1 - \sigma)(1 - \epsilon) \sum_{e' \in \{0, 1\}} p_{i,0,e'} L_{i,e'}(r, q) \]  

(5)

Note that unemployed renters pay 0 and may be evicted in (5). The solution to (5) induces \( \epsilon_{i,0}(r, q) \). The landlord chooses to evict \( (\epsilon_{i,0}(r, q) = 1) \) if expected discounted profits are negative because posting a new vacancy has zero net profit for landlord. That is, if:

\[
\sum_{e' \in \{0, 1\}} p_{i,0,e'} L_{i,e'}(r, q) < 0 \iff p_{i,0,1} \left( L_{i,1}(r, q) - L_{i,0}(r, q) \right) + L_{i,0}(r, q) < 0
\]

It is clear from above (6) that eviction is more likely for \( L \)-type renters because they have a lower job finding rate since \( p_{L,0,1} = 0.17 < p_{H,0,1} = 0.89 \) from Table 2.

A renter in a unit of quality \( q \) with constant rent \( r \) given the landlord’s optimal eviction choice \( \epsilon_{i,0}(r, q) \) has the following values:

\[ R_{i,1}(r, q) = y_i - \alpha + qE(Q) - r + \beta (1 - \sigma) \sum_{e' \in \{0, 1\}} p_{i,1,e'} R_{i,e'}(r, q) \]  

\[ + \beta \sigma \sum_{e' \in \{0, 1\}} p_{i,1,e'} V_{i,e'}^* \]  

\[ R_{i,0}(r, q) = qE(Q) + \beta (1 - \sigma)(1 - \epsilon_{i,0}(r, q)) \sum_{e' \in \{0, 1\}} p_{i,0,e'} R_{i,e'}(r, q) \]  

\[ + \beta (1 - (1 - \sigma)(1 - \epsilon_{i,0}(r, q))) \sum_{e' \in \{0, 1\}} p_{i,0,e'} V_{i,e'}^* \]  

(7)

Note that unemployed renters receive \( q \), pay 0, and may be evicted in (7). If evicted, the person becomes unhoused and searches next period obtaining \( V_{i,e}^* \).

Landlords post contracts over fixed rent \( r \) and quality \( q \) to which unhoused people direct their search to a submarket with tightness \( \theta \). The decentralized equilibrium allocations maximize unhoused utility in (8) subject to landlord participation in (9):

\[ V_{i,e}^* = y_i,e - \alpha + \max_{r \leq y_i,e - \alpha, q, \theta} \beta \left( \phi(\theta) \sum_{e' \in \{0, 1\}} p_{i,e,e'} R_{i,e'}(r, q) \right) \]  

\[ + (1 - \phi(\theta)) \sum_{e' \in \{0, 1\}} p_{i,e,e'} V_{i,e'}^* \]  

(8)
\[ s.t. \quad \kappa \geq \beta \psi(\theta) \left[ \sum_{e' \in \{0,1\}} p_{i,e,e'} L_{i,e'}(r,q) - c(q - f_i) \right], \tag{9} \]

Free entry requires (9) holds with equality. Given the presence of subsistence consumption \( \alpha \), a renter of type \( i \) can afford to pay at most rent \( r_i = y_i - \alpha \). The free entry condition can also be used to show that our earlier assumption that \( \epsilon_{i,1}(r,q) = 0 \) holds in equilibrium with a positive measure of housed renters (see Appendix C for a proof). The solution to (8) and (9) yields \((r_{i,e}, q_{i,e}, \theta_{i,e})\).

We can now define a steady-state equilibrium.

**Definition 1.** A steady-state competitive search equilibrium with constant rent contracts is given by

i. rents \( r_{i,e} \) on units of quality \( q_{i,e} \) and vacancy posting for those contracts with tightness \( \theta_{i,e} \) satisfy (8) and (9) given (4) through (7) as well as unmatched values, \( V_{i,e}^* \),

ii. eviction choice \( \epsilon_{i,0}(r,q) \) satisfies (5),

iii. a fixed point of the laws of motion over employment and housing \( \mu_{i,e'}^h(r_{i,k}, q_{i,k}) \) in (10) and \( \mu_{i,e'}^u \) in (11) for \( i \in \{H, L\}, \ e' \in \{0,1\}, \) and \( k \in \{0,1\} \) given by:

\[
\mu_{i,e'}^h(r_{i,k}, q_{i,k}) = \sum_{e \in \{0,1\}} p_{i,e,e'} (1 - \sigma) \left( 1 - \epsilon_{i,e}(r_{i,k}, q_{i,k}) \right) \mu_{i,e}^h(r_{i,k}, q_{i,k}) \\
+ p_{i,k,e'} \phi(\theta_{i,k}) \mu_{i,k}^u
\tag{10}
\]

\[
\mu_{i,e'}^u = \sum_{e \in \{0,1\}} p_{i,e,e'} \left\{ \sum_{j \in \{0,1\}} \left[ 1 - (1 - \sigma) \left( 1 - \epsilon_{i,e}(r_j, q_j) \right) \right] \mu_{i,e}^h(r_j, q_j) \right\} \\
+ \left( 1 - \phi(\theta_{i,e}) \right) \mu_{i,e}^u
\tag{11}
\]

iv. aggregate housing quality is given by:

\[
Q = \sum_{i \in \{H,L\}} \sum_{e \in \{0,1\}} \sum_{j \in \{0,1\}} \mu_{i,e}^h(r_{i,j}, q_{i,j}) q_{i,j}.	ag{12}
\]

The fixed point of the law of motion in (10) maps those who are housed and not evicted as well as those unhoused who find a rental (the right hand side) into those who are housed
(the left-hand side) while maintaining the fixed contract terms \((r_{i,k}, q_{i,k})\) corresponding to employment status, \(k \in \{0, 1\}\), when they first found their housing.

In the case without externalities (i.e. \(\mathcal{E}(Q) = 1\)), the expressions in equations (4) through (9) are independent of the distributions of people over housing and employment states. This means that the equilibrium is block recursive and we can calculate the equilibrium objects in (i) and (ii) separately from those in (iii) through (iv). With an externality, we need to know the distributions since they affect \(Q\), so our equilibrium is conditionally block recursive.\(^{11}\) That is, given \(Q\), we do not need the distributions in order to calculate (i) and (ii).

### 5.1 Inefficiency of Competitive Equilibrium

There are three ways that the decentralized equilibrium is inefficient. First, evictions may occur, whereas the social planner would never evict. Second, for a given type, the landlord’s expected discounted profits from posting a vacancy for the unemployed is lower than for the employed, which leads to better housing outcomes for the employed relative to the unemployed (whereas the social planner posts the same number of vacancies of the same quality for a given type of renter, independent of their employment status). Finally, the social planner takes the externality into account when creating housing, whereas an individual landlord ignores the effect of their quality investment on others.

Even without externalities, the first two inefficiencies can arise because the landlord cannot commit to keep an unemployed renter if evicting them would be more profitable. Constant rent contracts do not incentivize a landlord to keep an unemployed renter by raising future rent. One might wonder if variable contracts could eliminate evictions in equilibrium and restore equality of housing quality and supply for employed and unemployed households of a given type. For example, could landlords raise future rent enough to keep their expected discounted profits positive? In our calibrated economy, there is no room for variable contracts to improve because type-\(L\) households that face eviction pay their entire disposable income in rent; even if landlords would be willing to keep the tenant in exchange for higher future rent, paying more is simply not feasible for the renter (i.e. the consumption constraint \(C \geq \alpha\) binds, which implies \(r = y_L - \alpha\)).

In Appendix D we show that inefficient evictions are a general feature of a model with profit-seeking landlords, even without a binding constraint on the amount of rent that a tenant can pay in the future. Specifically, we allow an employed renter to work as many extra hours (i.e. overtime) as needed, so that any finite rental payment is feasible while maintaining non-negative consumption. In this setting, landlords can always set future rent

\(^{11}\)This concept of conditional block recursivity has been used in other settings, such as Chaumont and Shi [4].
high enough that they would be willing to keep an unemployed tenant, so long as the job-finding rate is positive. We call the lowest value of rent for which the landlord chooses not to evict an unemployed renter $r$. However, we assume that there is two-sided lack of commitment; the renter is allowed to walk away if the expected discounted value of seeking a new rental is higher than remaining in their current unit. Therefore, there is an upper bound on rent that the renter would be willing to pay in future dates when they are employed. We call this upper bound $\bar{r}$; if rent is raised any higher, then the tenant would move out and seek a new rental upon re-employment. Variable rate rent contracts can therefore avoid eviction if an unemployed tenant’s rent can be reset in the interval $[r, \bar{r}]$. If this interval is empty, then eviction occurs when the tenant becomes unemployed.

In Appendix D we show that our calibrated constant-rent equilibrium is also an equilibrium in the variable rate model if the $L-$type’s disutility from working enough overtime to avoid eviction is at least 2.5 times their normal income. We also provide a sufficient condition on the disutility of overtime to guarantee that an equilibrium with evictions exists in the extended model with overtime and variable rate contracts. We conclude that evictions are a general inefficiency due to lack of commitment by both landlords and renters.

6 Properties of Calibrated Economy

6.1 Parameter Estimates

We make the following functional form assumptions. The cost function is $c(q - f_i) = (q - f_i)/(1 + c_0)$. We use $M(U, V) = \frac{U \cdot V}{(U^{\nu} + V^{\nu})^{\frac{1}{\nu}}}$ which gives finding and filling rates of $\phi(\theta) = \frac{1}{(1 + \theta^{\nu})^{\frac{1}{\nu}}}$ and $\psi(\theta) = \frac{\theta}{(1 + \theta^{\nu})^{\frac{1}{\nu}}}$. This matching function gives non-constant elasticities of the finding and filling rates with respect to $\theta$, but remains bounded in $(0,1)$. We assume the functional form of the externality is given by $E(Q) = e^{\eta Q}$. This implies an externality semi-elasticity $\eta = \frac{d\log(E(Q))}{dQ}$.

Table 5 lists parameters that we calibrate directly from the data (the transition matrices, relative incomes, fixed costs for landlords, separation rate, and $L-$type rental burden). We are left with just four free parameters (the externality spillover, the elasticity of the matching technology, the parameter governing costliness of quality investment, and the cost of posting a vacancy), which we choose to best fit five additional moments, as listed in Table 9.
In Table 5, given we have a monthly model we set $\beta$ to match an annual real interest rate of around 4 percent. Our measures of $f_i$ are chosen to match the ratio operating costs to rental rates in Table 3 for high and low renters. The average duration of rental match is three years which pins down $\sigma$. The employment transition probabilities $p_{i,e,e'}$ comes from Table 2 using data from the Current Population Survey. We note that these estimates are consistent with type H individuals who are more likely to keep or find a job ($p_{H,e,1} > p_{L,e,1}$ for all $e$). Our estimates of job finding rates $p_{H,0,1} = 0.89$ implies an average duration of unemployment of 1.1 months for type H individuals and $p_{L,0,1} = 0.17$ implies an average duration of unemployment of 5.8 months for type L. Table 2 also pins down the fraction of each type of earner: $\mu_H = 0.9$ and $\mu_L = 0.1$. Given we take the subsistence level $\alpha = 1$ as a normalization, the rent burden for L types $r_L/y_L$ from Table 1 helps us pin down $y_L$. Specifically, we choose $y_L$ to be consistent with a binding constraint $r_L \leq y_L - \alpha$ in our equilibrium yielding $y_L = 2$. A binding constraint implies that type L is not saving, which is consistent with zero savings for the 25th percentile of renters we document in Table 1. Table 2 shows that monthly earnings for type H workers is roughly twice those for type L, so we set $y_H = 2 \cdot y_L$.

Note that we assumed that people could not save. However, in the calibrated economy this assumption is without loss of generality because the type L renter never has resources available to save after paying rent and subsistence consumption, while the type H renter would have no incentive to save as long as the real interest rate on savings was consistent with the discount rate (which requires $\beta(1 + r) \leq 1$).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.96$^{\frac{1}{12}}$</td>
</tr>
<tr>
<td>$f_H$</td>
<td>0.62</td>
</tr>
<tr>
<td>$f_L$</td>
<td>0.37</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$\frac{1}{36}$</td>
</tr>
<tr>
<td>$p_{L,1,1}$</td>
<td>0.57</td>
</tr>
<tr>
<td>$p_{L,0,1}$</td>
<td>0.17</td>
</tr>
<tr>
<td>$p_{H,1,1}$</td>
<td>0.96</td>
</tr>
<tr>
<td>$p_{H,0,1}$</td>
<td>0.89</td>
</tr>
<tr>
<td>$y_H$</td>
<td>4</td>
</tr>
<tr>
<td>$y_L$</td>
<td>2</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1</td>
</tr>
<tr>
<td>$\mu_L$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\mu_H$</td>
<td>0.9</td>
</tr>
</tbody>
</table>
Table 6: Parameters Calibrated in Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>η</td>
<td>0.138</td>
<td>spillover (Autor, et al.)</td>
<td>0.5</td>
<td>0.422</td>
</tr>
<tr>
<td>ν</td>
<td>1.384</td>
<td>vacancy rate (Census Bureau)</td>
<td>6.6</td>
<td>6.623</td>
</tr>
<tr>
<td>κ</td>
<td>0.295</td>
<td>eviction rate (Eviction Lab)</td>
<td>0.5</td>
<td>0.526</td>
</tr>
<tr>
<td>c₀</td>
<td>12.647</td>
<td>r_H/y_H (SCF)</td>
<td>1/3</td>
<td>0.458</td>
</tr>
<tr>
<td></td>
<td></td>
<td>r/q slope (RHFS)</td>
<td>0.45</td>
<td>0.37</td>
</tr>
</tbody>
</table>

The estimates of the remaining four parameters $b = (\eta, \nu, \kappa, c_0)$ of our overidentified model are listed in Table 9. All parameters are jointly dependent on the listed moments, we present which moments we believe help most in identifying the parameter. The externality semi-elasticity chosen to match the spillovers in Table 4 imply $\eta = 0.285$. Using the average elasticity of the filling rate to tightness for unhoused households in the market for new homes from Genosove and Han [10] helps pin down the match elasticity $\nu = 0.285$. The cost of posting housing vacancies $\kappa = 0.04$ helps pin down the eviction rate since it lowers the fraction of housed people who would be subsequently evicted. The rent burden for type $H$ helps pin down the cost parameter $c_0 = 7.73$. The rent-to-value slope is also important for helping to identify the cost parameter $c_0$.

In Table 7 we compute a local measure $\Lambda^*$ of the elasticity of our estimated parameters to a change in each of the model moments in Table 9 based on Andrews, et. al. [3] (hereafter AGS).\(^\text{13}\) Element $(i, j)$ of $\Lambda^*$ can be interpreted as the elasticity of our estimate of parameter

\(^\text{12}\) We replicate the policy experiment from Autor et al. [2] by comparing the baseline equilibrium to an equilibrium where 40 percent of renters of each type are randomly assigned to become rent-constrained, imposing that $r \leq 0.95\bar{r}_i$ where $\bar{r}_i$ is the average $r$ of the housed of type $i$ in the baseline stationary equilibrium. The rent ceiling reduces the quality chosen by these renters, which lowers total quality $Q$ and spills over to the unconstrained renters within the experiment. We interpret $q e^{\xi Q}$ as being proportional to the value from the housing. The spillover moment is then computed as the difference in mean value from housing for the unconstrained renters as a fraction of the difference in mean value from housing for the constrained renters, i.e. $\frac{4.047-4.017}{4.937-3.981}$ from the table below, computed at the calibrated equilibrium.

### Spillover Experiment Results

<table>
<thead>
<tr>
<th>Qualities</th>
<th>Baseline</th>
<th>Unconstrained</th>
<th>Constrained</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>2.005</td>
<td>1.97</td>
<td>1.97</td>
</tr>
<tr>
<td>mean $q e^{\xi Q}$</td>
<td>3.003</td>
<td>2.963</td>
<td>2.907</td>
</tr>
</tbody>
</table>

\(^\text{13}\) In Appendix G we present the calculation of $\Lambda^*$. Specifically, we begin by computing $\Lambda$ from AGS,

$$\Lambda = -(G'WG)^{-1}G'W$$

where $G = \mathbb{E}[\nabla_b \hat{g}(b)]$ is the $5 \times 4$ probability limit of the Jacobian of the vector of deviations of model
with respect to moment $j$. For example, we can interpret the upper-left element of Table 7 as implying that a 1 percent increase of $r_H/y_H$ would lead to a decrease of our estimate of $\eta$ of 0.44 percent. Table 7 suggests that our parameters are most sensitive to the match elasticity moment, and $\kappa$ is also very sensitive to the eviction rate.

Table 7: Sensitivity Matrix $\Lambda^*$

<table>
<thead>
<tr>
<th></th>
<th>$r_H/y_H$</th>
<th>Eviction rate</th>
<th>$r/q$ slope</th>
<th>Spillover</th>
<th>Vacancy Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>2.49</td>
<td>0.38</td>
<td>-4.27</td>
<td>-11.47</td>
<td>-0.42</td>
</tr>
<tr>
<td>$\nu$</td>
<td>-0.25</td>
<td>-0.96</td>
<td>1.64</td>
<td>4.91</td>
<td>0.03</td>
</tr>
<tr>
<td>$C_0$</td>
<td>1.84</td>
<td>0.24</td>
<td>-2.12</td>
<td>-5.19</td>
<td>-0.19</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>-0.08</td>
<td>-0.01</td>
<td>0.15</td>
<td>0.41</td>
<td>-0.09</td>
</tr>
</tbody>
</table>

6.2 Model Properties

We illustrate the equilibrium workings of the baseline model in the calibrated economy in Tables 8 through 11. Key for understanding the eviction decision and contract terms is the earnings prospects of our two types of renters that came from the CPS in Table 1. Specifically, the data suggests $P_{H,e,1} > P_{L,e,1}$ and $y_H > y_L$ so that type $H$ have higher expected income than type $L$ and shorter duration of unemployment. These “exogenous” income prospect differences induce differences in equilibrium rental terms (rental rates and qualities) where a high income renter pays 6 percent higher rates and receives substantially (77 percent) higher quality housing than a low earner evident in equation 8. This is consistent with the lower rent-to-quality ratios among low and higher earners. Importantly a high earner who is currently unemployed is offered a contract (albeit with slightly lower quality housing than her employed counterpart) while a low earner who is unemployed does not even receive a housing contract (i.e. is unhoused). Therefore, in equilibrium there are only three rental contracts $C = \{(r_{H,1}, q_{H,1}), (r_{H,0}, q_{H,0}), (r_{L,1}, q_{L,1})\}$ offered. Consistent with the contract offerings, Table 8 documents that the rental finding rate is higher for type $H$ than type $L$, where the expected duration of homelessness for a type $H$ person is just under two months while it is just under four months for an employed type $L$ person.

moments from data moments in $\hat{g}(b)$ we use in our simulated method of moments estimation procedure and $W$ is the probability limit of the weighting matrix, which we have simply taken to be the identity matrix. We then compute $\Lambda^*$ by multiplying each element of $\Lambda$ by $m_j b_i$ where $m_j$ is the corresponding data moment and $b_i$ the corresponding estimated parameter. $\Lambda^*$ then has the interpretation of a matrix of elasticities, and is hence invariant to the scale of the parameters and moments whereas $\Lambda$ has the interpretation of a derivative and is sensitive to the scaling of both the data moments and the model parameters.
Table 8: Calibrated Equilibrium

<table>
<thead>
<tr>
<th>Policies</th>
<th>Baseline</th>
<th>No-Spillovers ($\eta = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(r_{H,1}, q_{H,1})$</td>
<td>(1.833, 2.303)</td>
<td>(1.667, 2.57)</td>
</tr>
<tr>
<td>$(r_{H,0}, q_{H,0})$</td>
<td>(1.823, 2.292)</td>
<td>(1.667, 2.566)</td>
</tr>
<tr>
<td>$(r_{L,1}, q_{L,1})$</td>
<td>(1.0, 0.465)</td>
<td>(1.0, 0.573)</td>
</tr>
<tr>
<td>$(r_{L,0}, q_{L,0})$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$\epsilon_{H,0}(r_{H,1}, q_{H,1})$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\epsilon_{H,0}(r_{H,0}, q_{H,0})$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\epsilon_{L,0}(r_{L,1}, q_{L,1})$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\epsilon_{L,0}(r_{L,0}, q_{L,0})$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\epsilon_{i,1}(r_{i,e}, q_{i,e})$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$(\theta_{H,1}, \phi(\theta_{H,1}))$</td>
<td>(0.56, 0.765)</td>
<td>(0.491, 0.875)</td>
</tr>
<tr>
<td>$(\theta_{H,0}, \phi(\theta_{H,0}))$</td>
<td>(0.539, 0.774)</td>
<td>(0.521, 0.864)</td>
</tr>
<tr>
<td>$(\theta_{L,1}, \phi(\theta_{L,1}))$</td>
<td>(3.482, 0.255)</td>
<td>(3.683, 0.259)</td>
</tr>
<tr>
<td>$(\theta_{L,0}, \phi(\theta_{L,0}))$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
</tr>
</tbody>
</table>

Notes: The rows labeled “newborn CE$_L$” and “newborn CE$_H$” represent the percentage change in consumption during each period that would make an agent indifferent between being born into the baseline economy and the economy described in that column. The mathematical definition can be found in equation (37) of Appendix F.3.

The differences in earnings prospects between type $H$ and $L$ have important implications for equilibrium eviction $\epsilon_{i,e}(r, q)$ across the different contracts. Table 8 documents that landlords do not evict a type $H$ renter who becomes unemployed (i.e. $\epsilon_{H,0}(r, q) = 0$ since they will soon be back to paying a high rent) but to evict a type $L$ renter who becomes unemployed (i.e. $\epsilon_{L,0}(r, q) = 1$ since it will take a long time before they will be paying back their low rent). All types who are employed are not evicted in equilibrium (i.e. $\epsilon_{i,1}(r, q) = 0$).

---

14 Calibration table for no-externality:

Table 9: Parameters Calibrated in Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>1.819</td>
<td>spillover (Autor, et al.)</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.102</td>
<td>vacancy rate (Census Bureau)</td>
<td>6.6</td>
<td>6.554</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>8.192</td>
<td>eviction rate (Eviction Lab)</td>
<td>0.5</td>
<td>0.531</td>
</tr>
<tr>
<td>$C_0$</td>
<td></td>
<td>$r_{H}/y_{H}$ (SCF)</td>
<td>1/3</td>
<td>0.409</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$r/q$ slope (RHFS)</td>
<td>0.45</td>
<td>0.372</td>
</tr>
</tbody>
</table>
Table 10: Stationary measure $\mu_{i,e}^j$ for baseline

<table>
<thead>
<tr>
<th>Housing state $j$</th>
<th>Employed $e = 1$</th>
<th>Unemployed $e = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{L,e}^h(r_{L,1}, q_{L,1})$</td>
<td>0.007</td>
<td>0.005</td>
</tr>
<tr>
<td>$\mu_{L,e}^h(r_{L,0}, q_{L,0})$</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$\mu_{L,e}^u$</td>
<td>0.021</td>
<td>0.066</td>
</tr>
<tr>
<td>$\mu_{L,e}^h(r_{H,1}, q_{H,1})$</td>
<td>0.795</td>
<td>0.036</td>
</tr>
<tr>
<td>$\mu_{L,e}^h(r_{H,0}, q_{H,0})$</td>
<td>0.036</td>
<td>0.002</td>
</tr>
<tr>
<td>$\mu_{L,e}^u$</td>
<td>0.03</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 10 presents the equilibrium stationary cross-sectional distribution $\mu_{i,e}^j$ of the population of housed and unhoused $j \in \{h, u\}$ renters of type $i \in \{H, L\}$ of employed and unemployed $e \in \{0, 1\}$ households. While the fractions of each type $\mu_H = 0.9 = \sum_{e,j} \mu_{H,e}^j$ and $\mu_L = 0.1 = \sum_{e,j} \mu_{L,e}^j$ from the data in Table 5 are exogenously pinned down, the distribution within type is endogenous. The table makes clear that while there are only 10 percent of low earning renters in the population we focus on, they account for a large fraction of the homeless $\sum_{i,e} \mu_{i,e}^u$. Specifically, they account for $\frac{\sum_{e} \mu_{L,e}^u}{\sum_{i,e} \mu_{i,e}^u} = 65.4$ percent of below median income unhoused renters. Note further that some measure of employed households can be unhoused; these are those who either did not find housing because of finding rates between roughly one quarter and one half in Table 8 or because of exogenous separation.

Housing Spillovers

While our baseline uses the point estimates of Autor, et al [2] to calibrate the externality, their confidence interval cannot exclude the no spillover case. It is therefore useful to consider how spillovers affect quality and housing rates in our equilibrium, which we do by comparing the baseline to the no-spillover economy where we set $\eta = 0$ in Table 8.

As one might expect positive spillovers raise housing quality (and hence rents) but for our “local” deviation away from $\eta = 0.285$ we do not find a change in the eviction pattern. Table 8 also documents that finding rates fall and more people go homeless without positive spillovers. There are also important differences between households of different types due to the externality. Specifically, quality is raised for type $L$ but lowered for type $H$ in the no-spillovers equilibrium relative to our baseline (i.e. the quality differential is exacerbated by the externality). Further, while welfare falls for both types, it falls more for type $H$ ($-28$ percent) than for type $L$ ($-25$ percent) in consumption equivalent terms. Thus, housing

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$^{15}$These are calculated in Table 8. Consumption equivalent welfare gains/losses are defined throughout this paper as the percentage change in consumption in each period that would leave a person indifferent between being born into the baseline economy versus one with a different set of parameters or policies. The mathematical definitions for each table are found in Appendix F.
quality externalities are calibrated to be quite large and can widen the gap between rich and poor renters.

**Local Comparative Statics**

Table 19 in Appendix H presents an expanded version of the 5x4 Jacobian $G$ matrix used in the AGS sensitivity matrix $A$ evaluated at the estimated parameter values. Independently, it provides a way to understand how local deviations of our parameters affect model moments presented in Table 9. Here we simply describe the qualitative effects of changes in the parameters on our moments of interest: eviction rates, rental burdens, rent-to-quality ratio, finding rates, and total housing quality. Given we consider only local deviations, we find that the type $L$ budget constraint $r \leq y_L - \alpha$ continues to bind and hence focus on the rent burden of type $H$.

First, consider changes in landlord costs. As the flow cost $f_L$ increases, the $L$-type renters cannot afford a landlord passthrough of cost to rental payments so they receive lower quality and worse finding rates. The lower finding rates result in a lower eviction rate as there are fewer housed $L$-types to begin with. By contrast, as $f_H$ increases the $H$-types can afford the passthrough to substantially higher rental payments. This supports their finding rate, which rises, but also increases their rent burden. Total quality rises as more $H$-types are housed, which increases the marginal benefit of quality and hence they increase their quality choice. As the posting cost $\kappa$ increases, fewer $L$-types are housed as the finding rates fall, leading to lower eviction rates. As before, $H$-types can afford the higher rental payments so their finding rate increases, bringing total housing quality up. As the cost $C_0$ of quality rises, $H$-types respond by lowering their quality choices which results in a lower housing quality.

Next, consider changes to renter income $y_i$ and its stochastic process $p_{i,e,e'}$. For the $L$-type renters, their binding rent constraint before and after the new income implies that all new income goes to rent payments. Hence, increasing $y_L$ allows them to receive higher finding rates and higher quality. The higher finding rates also imply more $L$-types are housed, and hence both total housing quality and finding rates increase. The rent-to-quality slope falls because the $r/q$ rises for the $L$-types. By contrast, $H$-type renters do not have a binding rent constraint, so increasing their income $y_H$ reduces their rent burden and increases their consumption but does not lead to any changes in equilibrium rental market choices or allocations. As $p_{H,1,1}$ rises, the $H$-type finding rate increases leading to more housing quality. Their per-period rent falls as they are employed, and hence pay rent, more frequently. The response to $p_{L,1,1}$ is similar but with some key differences. The finding rate and total quality rise as before, but for the $L$ types instead of offering less rent they receive higher quality. This is because the $L$-types have a binding rent constraint throughout. The eviction rate
rises for $L$-types because they are housed more often and because the change to the job keeping rate is small, but for sufficiently large changes the $L$ type is no longer evicted by the landlord. The effects of an increase to the job finding rate $p_{i,0,1}$ are similar for each type to the effect of an increase to their job keeping rate.

As the matching technology becomes more renter-friendly, with higher $\nu$, the finding rates for both $L$- and $H$-type renters increase. This increases housing quality, but also increases the eviction rate because more $L$-types are housed. Finally, as we increase neighborhood spillovers via the externality parameter $\eta$, the $H$-types increase the level of quality that they search over, and increase the rent they offer to maintain a high finding rate. As a result, neighborhood quality rises and the $H$-type rent burden rises. The rent-to-quality slope increases as the rise in $r/q$ for the $H$-types is much larger than the rise in $r/q$ for the $L$-types.

**Efficiency Comparison**

In Table 11 we compare the equilibrium allocation of quality and rental finding rates between our decentralized equilibrium and that chosen by the social planner not subject to the commitment friction. Recall that there is a fundamental difference in eviction rates between the planner’s problem (i.e. no one is evicted independent of type and employment status since all matches have positive surplus) while in the decentralized equilibrium type $L$ unemployed renters are evicted. Since employment status does not matter for the social planner, individual quality does not depend on employment status (i.e. $q_{i,sp}$) while individual quality in the decentralized equilibrium does depend on employment status (i.e. $q_{i,e}$). Since the planner internalizes the positive externality, they choose uniformly higher quality than the decentralized outcome, especially so for type $L$ households. This quality dominance chosen by the planner implies a nearly 16 percent higher total quality level $Q^{sp}$ than its decentralized counterpart $Q$. Importantly, there are vastly different fractions of type $L$ housed in the two allocations. Specifically, is 0.953 in the planner’s allocation while it is 0.124 in the decentralized equilibrium.
Table 11: Allocations in Planner’s and Competitive Equilibrium

<table>
<thead>
<tr>
<th>Variable</th>
<th>Planner</th>
<th>Competitive Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>2.30</td>
<td>1.988</td>
</tr>
<tr>
<td>$(q_{H,1}, \theta_{H,1}, \phi(\theta_{H,1}))$</td>
<td>(3.742,0.31743,0.874)</td>
<td>(2.303,0.56,0.765)</td>
</tr>
<tr>
<td>$(q_{H,0}, \theta_{H,0}, \phi(\theta_{H,0}))$</td>
<td>(3.742,0.31743,0.874)</td>
<td>(2.292,0.539,0.774)</td>
</tr>
<tr>
<td>$(q_{L,1}, \theta_{L,1}, \phi(\theta_{L,1}))$</td>
<td>(3.492,0.32835,0.869)</td>
<td>(0.465,3.482,0.255)</td>
</tr>
<tr>
<td>$(q_{L,0}, \theta_{L,0}, \phi(\theta_{L,0}))$</td>
<td>(3.492,0.32835,0.869)</td>
<td>$(0,\infty,0)$</td>
</tr>
<tr>
<td>L-type frac housed</td>
<td>0.969</td>
<td>0.122</td>
</tr>
<tr>
<td>H-type frac housed</td>
<td>0.969</td>
<td>0.965</td>
</tr>
</tbody>
</table>

Notes: Columns list the allocations of each variable from the planner’s solution and the competitive equilibrium. The rows labeled “i-type frac housed” are defined to be $\mu_i(i)$. These differences in allocations translate to large differences in aggregate discounted social surpluses. In Table 12 we calculate the losses from using the competitive equilibrium allocations of housing quality, tightness, and eviction decisions rather than the planner’s optimal choices. We can then perform a simple accounting exercise to decompose how far the competitive allocation is from the efficient one both due to lack of commitment and to the externality. The row labeled “Baseline $Q$” reports the loss in steady-state aggregate social surplus, relative to the planner’s optimum, using the tightnesses and qualities from the competitive equilibrium and assuming that matches are destroyed for type $L$ tenants whenever they lose their jobs. The row labeled “Planner $Q$” is similar, except that we fix the externality term at its value from the planner’s allocation ($\mathcal{E}(Q) = \mathcal{E}(Q^{opt})$). This calculation isolates the loss in welfare from the competitive equilibrium’s lack of commitment from the difference in the externality term. We find that three quarters of the loss ($-13.8$ percent) from the competitive equilibrium is due to lack of commitment, while another 4.5 percent is due to the externality ($-18.3$ percent in total).

Table 12: Welfare Loss From Competitive Equilibrium Relative to Planner Allocation

<table>
<thead>
<tr>
<th>Allocation</th>
<th>Aggregate Welfare Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline $Q$</td>
<td>-50.9%</td>
</tr>
<tr>
<td>Planner $Q$</td>
<td>-45.2%</td>
</tr>
</tbody>
</table>

Notes: The mathematical definitions can be found in equations (39) and (38) of Appendix F.3.

7 Quantitative Policy Evaluation

The inefficiency of the competitive equilibrium motivates us to consider possible policy responses, which may bring allocations closer to the planner’s solution and raise welfare. We
first consider restrictions on evictions, which is a direct attempt to overcome the problem that landlords cannot commit to keep an unemployed renter if eviction delivers higher profits. This reduces evictions but also expected profits for landlords and therefore reduces supply ex-ante. We then consider a subsidy to landlords that incentivizes them to keep unemployed renters who would otherwise be evicted by ensuring that the expected discounted profits are positive, even when the renter cannot pay today. In the next section we consider the effect of eviction policies in dealing with a crisis (i.e. outside of the steady state).

7.1 Eviction Policies

As evident in the previous section, unlike the social planner’s solution, landlords evict low income types since when unemployed they have a lower job finding rate and earn less (so once employed the landlord still cannot garner enough rent to cover the period of loss). Therefore, we consider if an eviction moratorium (to implement one part of the planner’s solution) is optimal in a decentralized competitive search environment. More generally, we ask if some restrictions on eviction are in fact optimal.

Specifically, a landlord who wants to evict a delinquent renter is allowed to do so with probability \( \lambda \in [0, 1] \). The prior subsection set \( \lambda = 1 \) while an eviction moratorium corresponds to \( \lambda = 0 \) (which effectively imposes landlord commitment). A policy maker who sets \( \lambda \) trades off two forces: (i) increased social surplus from maintaining a match arising from a low \( \lambda \); (ii) lower landlord profits (hence lower quality and/or vacancies) if landlords can’t evict an unemployed person arising from a low \( \lambda \).

Figure 2 illustrates the welfare effects (in consumption equivalents) for an unhoused employed type \( L \) renter from restricting evictions. Specifically, if there are no restrictions (i.e. \( \lambda = 1 \)), utility is lower than if there is some degree of restrictions. In fact, as the example shows, utility is maximized at \( \lambda = 0.42 \). On the other hand, starting at \( \lambda = 0.19 \) down to \( \lambda = 0 \), an unhoused employed \( L \) type person is so “costly” to a landlord that they do not find a rental unit. For \( \lambda > 0.19 \) the employed type \( L \) subsistence constraint \( c_{L,1} = y_L - \alpha - r_L \) is binding implying \( r_L = 1 \). The binding constraint implies the landlord cannot recoup a future higher rental rate than \( r_L=1 \). Thus, Figure 2 illustrates that some restrictions on eviction are optimal since eviction destroys matches with positive social surplus but a full out eviction moratorium means all type \( L \), both employed and unemployed, cannot find rental units.

Figure 3 illustrates the properties of the competitive search equilibrium across \( \lambda \) to com-

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16 As in Abramson [1] this probability captures the strength of tenant protections against evictions.
17 In contrast type \( H \) do not face a binding constraint throughout \( \lambda \in [0, 1] \) and pay approximately 5 percent more than the type \( L \) rent.
Figure 2: Unhoused Employed Low-type Renter Value

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{unhoused-employed-l-type-fig}
\caption{Unhoused Employed Low-type Renter Value}
\end{figure}

Notes: Black line: Consumption equivalent welfare relative to baseline ($\lambda = 1$) as defined in equation (40) in Appendix F.3. Peak at $\lambda = 0.42$ (red line). Blue line: permanently unhoused consumption equivalence with $\theta_{L,1} = 0$ starting at $\lambda = 0.19$.

plement the previous figure. The top panel shows that quality-to-rental price for low type people drops with more restrictions on the ability to evict. The middle panel shows that rental finding rates drop ($\phi(\theta_L)$ falls since $\theta_L$ falls) as evictions are restricted. Finally, the bottom panel shows that the total amount of neighborhood quality rentals $Q$ is non-monotonic. This arises because while individual quality $q$ (intensive margin) falls with eviction restrictions, the number of evictions (extensive margin) is also falling so it is a horse race between the two margins. The latter two outcomes provide an example of the unintended consequences of eviction restrictions similar to the unintended consequences of firing costs in Hopenhayn and Rogerson [11]; eviction restrictions which lower landlord profitability can result in less rental vacancies just as firing costs lower firm profitability resulting in higher unemployment.

Figure 4 illustrates the effect of a binding constraint on rental payments implied by the subsistence consumption requirement faced by all households. In particular, we consider the consequences of relaxing the constraint (in particular we set $\alpha = 0.9$ rather than $\alpha = 1$ as in the baseline calibration). The top right panel makes clear that reducing subsistence consumption means that the $L$ type person can afford to pay the landlord a higher rent when employed. In that case, the landlord is able to recoup more profits during type $L$ employment spells. Therefore, holding $\lambda$ constant, there would be a large increase in vacancy
creation and quality investment by landlords. Optimal policy responds by reducing $\lambda$, which reduces evictions relative to the baseline $\alpha = 1$ case. In both cases, the $L$-type renter has a large welfare gain for some interior $\lambda$ values, as evidenced by the top-left panel. The more constrained ($\alpha = 1$) $L$-type renter has a maximum welfare gain of 12 percent at $\lambda = 0.44$ while the less constrained ($\alpha = 0.9$) $L$-type gains 34 percent at $\lambda = 0.22$. The bottom-right panel shows that the equilibrium $Q$ value increases for these $\lambda$ levels, which passes through to the $H$-type renter through the externality, implying a Pareto improvement to welfare. $Q$ rises to a maximum of 1.99 for $\alpha = 1$ and 2.00 for $\alpha = 0.9$, which increases the $H$-type unhoused employed welfare by 0.1 and 0.2 percent.

### 7.2 Rent Subsidies

Instead of a policy restricting evictions as in the previous section, here we consider subsidies paid to landlords that lower the eviction rate. Specifically, we consider a policy that pays landlords a subsidy, $S$, whenever their $L$-type tenant is unemployed. This is financed on a flow basis by taxing the high-type individuals who are employed an amount $T$ given by:

$$T = \frac{\mu_L^h}{\mu_H^h + \mu_H^u} S. \tag{13}$$
We are only subsidizing the $L$—types because the idea is to subsidize enough that the landlord doesn’t evict. We are only taxing the $H$—type because the $L$—type cannot afford to pay any taxes and still consume above the subsistence threshold.

In Figure 5, we plot the effect of such a subsidy on the welfare of each type at birth as well as the expected welfare of a newborn before their type is drawn.\(^{18}\) We see that welfare is increasing for the $L$—type as we increase the subsidy, especially after $S = 0.12$, which is the point at which landlords no longer evict an unemployed $L$—type tenant. Despite being taxed, $H$—type welfare is increasing as the subsidy increases because their gains from housing spillovers dominate their lost utility from lower consumption. Hence, it is actually possible for a subsidy to be Pareto improving in the presence of housing spillovers, as was the case a partial eviction moratorium. The top-right panel shows that $H$—type welfare increases by 3 percent at $S = 0.25$, despite financing the subsidy through a tax of $T = 0.02$. This Pareto improvement entirely comes through the externality. Without the externality, i.e. $\eta = 0$, the $H$—type value strictly declines in $S$ in the top right panel due to the tax. However, the newborn value continues to strictly increase in $S$ over the range of values we consider.

Since the above experiment has potential costs and benefits, we also compute the same

\(^{18}\)We assume everyone is born unhoused and with an employment status drawn from the stationary distribution implied by their type-specific Markov chain.
equilibrium objects’ response to the subsidy policy setting the externality parameter $\eta = 0$ in Figure 5, which we can’t reject at the 5 percent level using the estimates from Autor et al. [2]. Without housing spillovers, the effect of rental subsidies on the $L$-types is almost identical while the effect on the $H$-types switches from being positive on net to negative, since the neighborhood externality quality benefits vanish but the loss from taxation remains. Still, the value of a newborn before drawing a type rises with the subsidy even without housing spillovers, as the benefits to the $L$-types are quite large whereas the costs to the $H$-types are quite small (the tax is small because 90 percent of the population is type H).

To understand the changes in the equilibrium caused by the rental subsidy, Figure 6 shows rental market outcomes for the $L$–type individual as $S$ rises. Since the type $L$ renter continues to face a binding constraint, the subsidy has no effect on their payment $r$ but does incentivize the landlord to change his eviction decision evident in the top left panel. Of note, equilibrium quantities display a noticeable kink at $S = 0.12$, as the landlord eviction policy shifts discreetly from $\epsilon = 1$ to $\epsilon = 0$ at that value of $S$. Once the $L$-type is no longer evicted once they are employed, their value shoots up as successful housing matches become much longer in duration. Notice that the presence of externalities changes little in these outcomes, so the fact that a subsidy can be Pareto improving with externalities is coming through the fact that the H-type’s flow utility from housing jumps discreetly when the L-type is no longer

Notes: Discounted expected utility for newborn as subsidy to landlords rises. See equations (41) and (42) in Appendix F.3 for the appropriate calculation of consumption equivalent.
evicted, which offsets the fact that the tax on H-types required to fund the subsidy are also higher (the bottom right panel in Figure 6).

Figure 6: Effect of Subsidy on Rental Market

Notes: Rental market outcomes as subsidy to landlord rises.

8 Economic Crises

We now add aggregate uncertainty to our environment. In particular there are two aggregate states, \( s \in \{G, B\} \) where \( s = G \) corresponds to a baseline state like that parameterized above and \( s = B \) corresponds to a crisis state where there is a sudden spike in unemployment. The timing of our model with aggregate uncertainty matches exactly the timing without aggregate uncertainty, but the Markov process on employment states depends on the current aggregate state. Specifically, the job-finding rates \( p_{i,0,1}(s) \) and job retention rates \( p_{i,1,1}(s) \) are aggregate state dependent. The aggregate state itself evolves according to a Markov process with realization prior to point 1 in our timing.\(^{19}\)

Given the landlord and renter values conditional on matching which are described in Appendix E, the unhoused renter solves the following:

\(^{19}\)We solve the model using techniques as in Krusell and Smith [15]. See Appendix F for more details.
\[
V_{i,e}^*(s, \mu) = y_{i,e} - \alpha + \max_{r \leq y_i - \alpha, q, \theta} \beta \mathbb{E}_{s'} |s\left[ \phi(\theta) \left( \sum_{e' \in \{0,1\}} p_{i,e,e'}(s) R_{i,e'}(r, q; s', \mu') \right) \\
+ (1 - \phi(\theta)) \left( \sum_{e' \in \{0,1\}} p_{i,e,e'}(s) V_{i,e'}^*(s', \mu') \right) \right] \]

s.t.

\[
\kappa \geq \beta \psi(\theta) \mathbb{E}_{s'} |s\left[ \sum_{e' \in \{0,1\}} p_{i,e,e'}(s) L_{i,e'}(r, q; s') - c(q - f_i) \right],
\]

As discussed above, state \( s = G \) is parameterized as in the benchmark above which appears again in Table 13. We model the bad state \( s = B \) after the observed job finding rates observed during the Covid-19 pandemic. We assume that state \( s = G \) is very persistent (expected duration is 100 years) while the crisis state \( s = B \) is transitory (expected duration is 4 months). In this environment, we consider three choices for eviction policies. Specifically, we allow for the eviction success rate to be aggregate-state dependent, \( \lambda(s) \), and consider a no-moratorium policy \( (\lambda(G), \lambda(B)) = (1, 1) \), crisis moratorium in a \( B \) state but not in a \( G \) state \( (\lambda(G), \lambda(B)) = (1, 0) \), and full moratorium \( (\lambda(G), \lambda(B)) = (0, 0) \).

Table 13: Aggregate Uncertainty Parameterization

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covid 19 Calibration</td>
<td></td>
</tr>
<tr>
<td>((p_{L,0.1}(G), p_{H,0.1}(G)))</td>
<td>(0.17, 0.89)</td>
</tr>
<tr>
<td>((p_{L,1.1}(G), p_{H,1.1}(G)))</td>
<td>(0.57, 0.96)</td>
</tr>
<tr>
<td>((p_{L,0.1}(B), p_{H,0.1}(B)))</td>
<td>(0.09, 0.80)</td>
</tr>
<tr>
<td>((p_{L,1.1}(B), p_{H,1.1}(B)))</td>
<td>(0.45, 0.91)</td>
</tr>
<tr>
<td>(Pr(s' = G</td>
<td>s = G))</td>
</tr>
<tr>
<td>(Pr(s' = G</td>
<td>s = B))</td>
</tr>
</tbody>
</table>

Table 14 lists the equilibrium outcomes with aggregate uncertainty. We will start with the two columns under “No Moratoria”, the effect of a crises on equilibrium variables is can be seen by comparing the \( s = G \) and \( s = B \) columns. While type \( H \) individuals are barely affected by the crisis (their finding rate falls marginally), the finding rate and quality investments for the type \( L \) units fall dramatically. In addition, the type \( L \) tenants who become unemployed are immediately evicted in the crisis. As seen from the black solid line
in Figure 7, this leads to a deep decline in the share of type L people who are housed that persists for over eight months after the crisis ends.

The next two columns in Table 14 shows the effect of an eviction moratorium during the crisis that is removed once the economy recovers. The moratorium results in a shutdown of new matches for $L$-types during the crisis, so that the finding rate falls from 0.26 to 0.0 (rather than to 0.13 without a moratorium). For an $L$–type who finds themselves unhoused during the crisis, an eviction moratorium makes it impossible to find new housing. However, the blue dashed line in Figure 7 shows that the crisis moratorium eliminates the sharp fall in housing during the crisis, since the unemployed cannot be immediately evicted (the housing rate falls only due to exogenous separations). In terms of welfare, the gain from maintaining matches outweights the decline in finding rates and the crisis moratorium raises welfare by 1.8 percent for type $L$ people (see Table 15). While type $H$ people are mostly unaffected by the eviction policy, spillovers from the quality externality lead them to gain from the crisis moratorium as well.

While a temporary eviction moratorium during the crisis is Pareto improving, a permanent moratorium is a bad policy for the same reason as in the steady-state model. The last two columns of Table 14 show that a permanent moratorium completely shuts down rental markets for type $L$ rentals, leading to an ever declining share who are housed in Figure 7. This reduces welfare for type $L$ people dramatically (by 43.6 percent in Table 15) and even reduces welfare for the type $H$ people through the externality.

To demonstrate the effect of the aggregate state-dependent moratorium policy, we plot the response of the fraction of $L$-type renters housed during a 3 period crisis. In Figure 7 we plot the responses under a no-moratorium policy (i.e. $(\lambda(G),\lambda(B)) = (1,1)$) and
under a crisis-moratorium policy (i.e. \((\lambda(G), \lambda(B)) = (1, 0)\)). The difference in outcomes for the L-type renters is stark. Without the moratorium policy 50 percent of housed L-type renters are evicted the end of the crisis, and as many as 68 percent are unhoused immediately following the crisis. Under the state-dependent moratorium policy, however, the L-type renters are allowed to remain housed throughout the crisis, with all separations during this time occurring exogenously at rate \(1 - \sigma\). Evictions are allowed again after the crisis has concluded, starting in period 3, and a large mass of renters is evicted at this time. By the beginning of period 3, the first post-crisis period, 91 percent of the L-type renters remain housed relative to 32 percent without the moratorium policy. While the lifting of the moratorium after the crisis leads to a later rise in evictions, causing the relative fraction housed to fall to 40 percent in period 4, there are large gains to L-type renters during the crisis itself. Overall, many more L-type renters are able to remain housed throughout the crisis under the crisis moratorium policy. For sake of comparison, under a permanent moratorium policy (i.e. \((\lambda(G), \lambda(B)) = (0, 0)\)) the L-type rental market shuts down and while the moratorium prevents a sudden wave of evictions, it results in disastrous long-run consequences. By period 15 under the permanent moratorium, the fraction of L-types housed is 66 percent of the baseline steady-state fraction and the fraction housed under this policy eventually converges to a new steady state without any housed L-type renters.
Table 15: Aggregate Welfare Comparison

<table>
<thead>
<tr>
<th></th>
<th>Crisis Moratoria</th>
<th>Full Moratoria</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CE_L$</td>
<td>0.1%</td>
<td>-1.7%</td>
</tr>
<tr>
<td>$CE_H$</td>
<td>0.1%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Notes: Consumption equivalent calculations in equation (43) of Appendix F.3.

There can be Pareto-improvements to welfare by instituting temporary crisis-dependent eviction moratoria. $L$- and $H$-type households gain 1.8 percent and 2.1 percent, respectively, in welfare from the crisis-dependent eviction moratorium relative to the no-moratorium baseline. The losses from a permanent moratorium policy, for comparison, are very large: $L$-types suffers a welfare loss of 43.6 percent while even $H$-type renters suffer a loss of 0.1 percent.

9 Conclusion

We present an equilibrium theory of rental markets in which the quality and tightness of the rental market is endogenous. Our model is realistic enough to capture salient features of rental markets in lower-income neighborhoods, such as eviction rates, higher rent burdens and rent-to-quality for the lowest income tenants, and large neighborhood externalities that exacerbate housing inequality. However, it is stylized enough that we can fully characterize the socially optimal allocation of housing, which is starkly egalitarian - evictions never occur and the quality and supply of housing are independent of a person’s employment status or income.

Importantly, the model is a useful laboratory for considering the social desirability of eviction restrictions and rent support for unemployed people, during both normal economic conditions and crises. The model illustrates that there can be important unintended consequences of eviction moratoriums emanating from the supply side of the rental market; eviction restrictions to keep people in rentals, even if ex-post socially optimal, result in lower supply of both vacancies and quality of rentals. Policymakers who wish to reduce evictions for at-risk renters without distorting the supply of housing (either quality or quantity) should instead subsidize the rent of unemployed tenants by paying the landlord directly.

References


A Appendix: Data

We describe the variables and sample selection for the Survey of Consumer Finance, Current Population Survey, and Rental Housing Finance Survey.

A.1 Survey of Consumer Finance

The Board of Governors provides a cleaned version of the Survey of Consumer Finance that provides useful variables defined at the household level. We use the following:

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rent</td>
<td>Monthly rent spending on all housing</td>
</tr>
<tr>
<td>Liquid Assets</td>
<td>Value of checking and savings balances</td>
</tr>
<tr>
<td>Networth</td>
<td>Value of all real and financial assets less all debts</td>
</tr>
<tr>
<td>Income</td>
<td>Income from all sources</td>
</tr>
</tbody>
</table>

A.2 Current Population Survey

We have matched individuals from 2018 to 2019 from monthly interviews in the CPS using their household identifier, individual identifier, state of residence, sex, race, and age. We then used the following variables to classify individuals by renter status, select working-age
renters with below-median earnings, calculate average earnings, and estimate transition rates between employment statuses:

Table 17: CPS Variables

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing type (hetenure)</td>
<td>Whether person owns housing, rents, or neither</td>
</tr>
<tr>
<td>Age (prtage)</td>
<td>Age of individual in years</td>
</tr>
<tr>
<td>Earnings (maximum value of prernwa)</td>
<td>Reference week’s earnings</td>
</tr>
<tr>
<td>Employment (lfs)</td>
<td>Labor force status</td>
</tr>
</tbody>
</table>

A.3 Rental Housing Finance Survey

In order to calculate operating costs, which we map to $f_i$ in our model, we use the operating cost variable from the RHFS as well as our own imputation of interest expenses for any debt on the rental unit as well as property taxes on the rental unit.

The RHFS measure of operating costs includes utilities, insurance, landscaping, management company expenses, payroll expenses, maintenance, and security. We add to this an estimate of interest payments, which we compute using the RHFS information on mortgages. We first take the initial amount borrowed, which is given in the RHFS. We then take the date when the first mortgage was taken, which is given in 3-10 year ranges, and use the average interest rate during the period of origination to calculate the average interest payment in 2018 on the mortgage assuming a standard 30-year term. Finally, we add $\frac{1}{12}$ percent of the rental unit’s market value to approximate the monthly property tax cost.

B Appendix: Social Planner’s Problem

Recalling that the flow utility to a person of type $i$ in a housing unit of quality $q$ is $q \cdot \mathcal{E}(Q)$, this means that the total flow of utility from housing to society is simply $Q \mathcal{E}(Q)$. The social planner solves:
\[ W(Q, (\mu_{i,e}^j)_{j \in \{u,h\} \in \{H,L\}, e \in \{0,1\}}) = \max_{(q_{i,e}, \theta_{i,e})_{i \in \{H,L\} \times \{0,1\}}} Q \mathcal{E}(Q) \] 

\[ + \sum_{i \in \{H,L\}} \left[ \mu_{i,1}^h (y_i - \alpha - f_i) + \mu_{i,0}^h (-f_i) + \mu_{i,1}^u (y_i - \alpha) + \mu_{i,0}^u (0) \right] \]

\[ - \sum_{i \in \{H,L\}, e \in \{0,1\}} \left[ \kappa + c(q_{i,e})\psi(\theta_{i,e}) \right] \theta_{i,e}^{-1} \cdot \mu_{i,e}^u \]

\[ + \beta \cdot W(Q', (\mu_{i,e'}^j)_{j \in \{u,h\} \in \{H,L\}, e' \in \{0,1\}}) \]

s.t.

\[ Q' = (1 - \sigma) \cdot Q + \sum_{e' \in \{0,1\}} \mu_{i,e}^u \cdot \phi(\theta_{i,e}) \cdot q_{i,e} \] 

\[ \mu_{i,1}^{h'} = p_{i,1,1} \left[ \mu_{i,1}^h (1 - \sigma) + \mu_{i,1}^u \phi(\theta_{i,1}) \right] + p_{i,0,1} \left[ \mu_{i,0}^h (1 - \sigma) + \mu_{i,0}^u \phi(\theta_{i,0}) \right] \] 

\[ \mu_{i,0}^{h'} = p_{i,1,0} \left[ \mu_{i,1}^h (1 - \sigma) + \mu_{i,1}^u \phi(\theta_{i,1}) \right] + p_{i,0,0} \left[ \mu_{i,0}^h (1 - \sigma) + \mu_{i,0}^u \phi(\theta_{i,0}) \right] \]

\[ \mu_{i,1}^{w'} = p_{i,1,1} \left[ \mu_{i,1}^h \sigma + \mu_{i,1}^u \left( 1 - \phi(\theta_{i,1}) \right) \right] + p_{i,0,1} \left[ \mu_{i,0}^h \sigma + \mu_{i,0}^u \left( 1 - \phi(\theta_{i,0}) \right) \right] \]

\[ \mu_{i,0}^{w'} = p_{i,1,0} \left[ \mu_{i,1}^h \sigma + \mu_{i,1}^u \left( 1 - \phi(\theta_{i,1}) \right) \right] + p_{i,0,0} \left[ \mu_{i,0}^h \sigma + \mu_{i,0}^u \left( 1 - \phi(\theta_{i,0}) \right) \right] \]

### C Appendix: Proof of No Eviction for employed renters

This is implied by the free entry condition. Consider a renter of type \( i \) who is in a rental match with a landlord in the stationary equilibrium, and let the terms of that rental contract be \((r, q)\). Given the landlord’s outside option in equilibrium is zero, eviction occurring when the renter is employed implies \( \sum_{e' \in \{0,1\}} p_{i,e,e'} L_{i,e'}(r, q) \leq 0 \). However, letting \( \tilde{e} \) be the employment state the renter was in when they successfully searched for the renter, \( \theta \) the equilibrium tightness of the submarket, and given that the landlord chose to enter, the free entry condition reads:

\[ \kappa = \beta \psi(\theta) \left[ \sum_{e' \in \{0,1\}} p_{i,\tilde{e},e'} L_{i,e'}(r, q) - c(q - f_i) \right]. \]

Since \( \kappa > 0, c(q - f_i) > 0, p_{i,1,1} > p_{i,0,1}, \) and \( L_{i,1}(r, q) > L_{i,0}(r, q) \), the free entry condition
implies:
\[ 0 < \frac{\kappa}{\beta \psi(\theta)} + c(q - f_i) = \sum_{e' \in \{0,1\}} p_{i,e,e'} L_{i,e'}(r, q) \leq \sum_{e' \in \{0,1\}} p_{i,1,e'} L_{i,e'}(r, q). \]

Hence, the result holds as the landlord strictly prefers continuing the match over her outside option.

**D Appendix: Variable Rate Contracts**

Here we focus on whether a variable rate contract can help solve the two sided lack of commitment problem leading to inefficient (relative to the social planner’s allocation) evictions. Since constant rent contracts do not incentivize a landlord to keep an unemployedrenter by raising future rent, here we consider if variable contracts could eliminate evictions in equilibrium and restore equality of housing quality and supply for employed and unemployed households of a given type. Specifically, can landlords raise future rent to provide themselves the incentive to keep an unemployed renter?

While this was not possible in our calibrated economy because there was no room for variable contracts to type-$L$ households because they ran up against a binding non-negativity constraint on consumption when employed, we now extend the model to allow employed renters to work as many extra hours (e.g. overtime) as needed, which incurs disutility, but makes any rental payment feasible while maintaining positive consumption. However, because the model features two-sided lack of commitment, tenants can walk away and search for a new rental if the cost of the variable rate contract is too high. Here we calculate the disutility of working overtime required for evictions to occur for our calibrated economy with constant rent contracts and further show that there is a generic set of parameters for which the $L$--type renter is evicted upon job loss.

**D.1 Extension of the Environment**

Fundamentally, the model is very similar to our baseline, except that we now assume that the income workers receive when employed comes from a labor supply choice. We assume that it is costless for a worker to generate $y_i$ units of income and that any hours worked in excess of what is required to generate $y_i$ delivers additional income at a unit wage, while causing disutility $\varphi(h^+)$, where $h^+ \geq 0$ and $\varphi'(h^+) \geq 1$.\(^{20}\) This “overtime” technology allows for

\(^{20}\)This nests the standard transferable utility model if $\varphi(h^+) = h^+$
non-negative consumption for any level of rent, i.e. \( y_i + h^+ - \alpha - r \geq 0 \) as long as \( h^+ \) is sufficiently high.

Landlords post contracts with initial rent and reset rent, \( r = (r, r^+) \) in submarkets based on \((i, e, q, r)\), which have tightness \( \theta_{i,e}(r, q) \). The cost of posting a vacancy is \( \kappa \) and quality investment is determined after matching, subject to cost \( c(q) \). We will assume that there is some \( \bar{q} \) such that \( \lim_{q \to \bar{q}} c(q) = \lim_{q \to \bar{q}} c'(q) = \infty \).

We will now find a \( \phi \) function for which our baseline calibrated equilibrium with constant rate contracts is also an equilibrium of the extended model and further show that there is a generic set of parameters for which evictions occur in equilibrium with variable contracts.

### D.2 No Evict Values

We start with the landlord problem after a match has occurred. We want to find the lowest rent \( r \) such that the landlord is indifferent between evicting and receiving 0 versus keeping and receiving rent \( r \). That is,

\[
p_{i,0,1} L_{i,1}(r, q) + p_{i,0,0} L_{i,0}(r, q) = 0
\]

where \( L_{i,e} \) are given in equations (4) and (5) for a given \( q \). To save on notation, define:

\[
\mathcal{L}_i(r) \equiv p_{i,0,1} L_{i,1}(r, q) + p_{i,0,0} L_{i,0}(r, q)
\]

\[
\Delta L_i(r) \equiv L_{i,1}(r, q) - L_{i,0}(r, q)
\]

\[
\Delta p_i \equiv p_{i,1,1} - p_{i,0,1}
\]

Then equations 4) and 5) into equation (20) yields

\[
\mathcal{L}_i(R) = p_{i,0,1} \left[ r - f_i + \beta (1 - \sigma) \left( p_{i,1,1} L_{i,1}(R, q) + p_{i,1,0} L_{i,0}(R, q) \right) \right] \\
+ p_{i,0,0} \left[ - f_i + \beta (1 - \sigma) \mathcal{L}_i(R) \right]
\]

Adding and subtracting \( \mathcal{L}_i(r) \) from \( p_{i,0,1} L_{i,1}(R, q) + p_{i,0,0} L_{i,0}(R, q) \) yields

\[
p_{i,1,1} L_{i,1}(R, q) + p_{i,1,0} L_{i,0}(R, q) + \mathcal{L}_i(R) - p_{i,0,1} L_{i,1}(R, q) - p_{i,0,0} L_{i,0}(R, q) \\
= \mathcal{L}_i(R) + \Delta p_i \Delta L_i(R)
\]
where we use the fact that
\[ p_{i,1,0} - p_{i,0,0} = (1 - p_{i,1,1} - (1 - p_{i,0,1})) = -(p_{i,1,1} - p_{i,0,1}) - \Delta p_i \] (26)

and the definitions in equations (21) through (23). Plugging equation (25) into equation (24) yields:

\[
\mathcal{L}_i(r) = p_{i,0,1}(r - f_i + \beta(1 - \sigma)[\mathcal{L}_i(r) + \Delta p_i \Delta L_i(r)]) + p_{i,0,0}(-f_i + \beta(1 - \sigma)\mathcal{L}_i(r))
\]
\[
= p_{i,0,1}r - f_i + \beta(1 - \sigma)\mathcal{L}_i(r) + \beta(1 - \sigma)p_{i,0,1}\Delta p_i \Delta L_i(r)
\] (27)

Again by equations (4) and (5) we know that equation (22) can be written:

\[
\Delta L_i(r) = \frac{r}{1 - \beta(1 - \sigma)\Delta p_i}
\] (28)

Plugging equation (28) into equation (27) yields

\[
\mathcal{L}_i(r) = p_{i,0,1}r - f_i + \beta(1 - \sigma)\mathcal{L}_i(r) + \beta(1 - \sigma)p_{i,0,1}\frac{\Delta L_i(r)}{1 - \beta(1 - \sigma)\Delta p_i}
\]
\[
= \frac{p_{i,0,1}r - (1 - \beta(1 - \sigma)\Delta p_i)f_i}{(1 - \beta(1 - \sigma))(1 - \beta(1 - \sigma)\Delta p_i)}
\] (29)

Then plugging equation (29) into equation (20) implies

\[
\mathcal{L}_i(r) = 0 \iff r = \left(\frac{1 - \beta(1 - \sigma)(p_{i,1,1} - p_{i,0,1})}{p_{i,0,1}}\right)f_i
\] (30)

Thus, if the renter agrees to pay \( r \geq \bar{r} \), the landlord will agree not to evict. Intuitively, \( \bar{r} \) is higher if the fixed cost of keeping a unit occupied the large, the job finding rate is small, or the job-keeping rate is small.

**D.3 No Renter Exit Values**

In this case we can re-write equation (6) as

\[
R_{i,1}(r, q) = \max_{h^+, h^+ + y_i - \alpha - r} h^+ - \varphi(h^+) + y_i - \alpha - r + q\mathcal{E}(Q)
\]
\[
+ \beta(1 - \sigma)\left[p_{i,1,1}R_{i,1}(r, q) + p_{i,1,0}R_{i,0}(r, q)\right]
\]
\[
+ \beta\sigma\left[p_{i,1,1}V^*_i + p_{i,1,0}V^*_i\right]
\]
and equation (7) as

\[ R_{i,0}(r, q) = qE(Q) + \beta(1 - \sigma)[p_{i,0,1}R_{i,1}(r, q) + p_{i,0,0}R_{i,0}(r, q)] \]

\[ + \beta \sigma [p_{i,0,1}V_{i,1}^* + p_{i,0,0}V_{i,0}^*] . \]

Defining the conditional expectations \( E_e[R_i] = [p_{i,e,1}R_{i,1}(r, q) + p_{i,e,0}R_{i,0}(r, q)] \) and \( E_e[V_i^*] = [p_{i,e,1}V_{i,1}^* + p_{i,e,0}V_{i,0}^*] \), we can rewrite equation (6) as

\[ R_{i,1}(r) = \begin{cases} 
  y_i - \alpha - r + qE(Q) + \beta(1 - \sigma)E_1[R_i] + \beta \sigma E_1[V_i^*] & \text{if } r \leq y_i - \alpha \\
  -\varphi(r + \alpha - y_i) + qE(Q) + \beta(1 - \sigma)E_1[R_i] + \beta \sigma E_1[V_i^*] & \text{if } r > y_i - \alpha
\end{cases} \quad (31) \]

since \( \varphi'(h^+) > 1 \) the marginal consumption benefit is outweighed by the marginal disutility from effort so that \( h^+ = r + \alpha - y_i \).

Given that we want to avoid eviction (which in our benchmark only occurs for type \( L \) agents), we want to find the highest rent \( \bar{r} \) such that an employed type \( L \) renter is indifferent between paying the higher rent and exiting to find a new rental in the case where \( r > y_L - \alpha \):

\[ R_{L,1}(\bar{r}, q) = V_{L,1}^* \quad (32) \]

Simple algebra on equation (7) yields

\[ R_{L,0}(\bar{r}, q) = \frac{qE(Q) + \beta(1 - \sigma)p_{L,0,1}R_{L,1}(\bar{r}, q) + \beta \sigma E_0[V_L^*]}{1 - \beta(1 - \sigma)p_{L,0,0}} \quad (33) \]

Plugging equation (33) into the bottom expression of (31) yields

\[ R_{L,1}(\bar{r}, q) = -\varphi(\bar{r} + \alpha - y_L) + qE(Q) + \beta \sigma E_1[V_L^*] \]

\[ + \beta(1-\sigma) \left[ p_{L,1,1}R_{L,1}(\bar{r}, q) + p_{L,1,0} \cdot \left\{ \frac{qE(Q) + \beta(1 - \sigma)p_{L,0,1}R_{L,1}(\bar{r}, q) + \beta \sigma E_0[V_L^*]}{1 - \beta(1 - \sigma)p_{L,0,0}} \right\} \right] \]

which after tedious algebra can be written

\[ R_{L,1}(\bar{r}, q) [1 - \beta(1 - \sigma)] [1 - \beta(1 - \sigma)\Delta p_L] \]

\[ = -\varphi(\bar{r} + \alpha - y_L) (1 - \beta(1 - \sigma)p_{L,0,0}) + [1 - \beta(1 - \sigma)\Delta p_L] \cdot qE(Q) \]

\[ + \beta(1-\sigma)p_{L,1,0} \cdot \beta \sigma E_0[V_L^*] + (1 - \beta(1 - \sigma)p_{L,0,0}) \beta \sigma E_1[V_L^*] \]

\[ \text{and equation (7) as} \]

\[ R_{i,0}(r, q) = qE(Q) + \beta(1 - \sigma)[p_{i,0,1}R_{i,1}(r, q) + p_{i,0,0}R_{i,0}(r, q)] \]

\[ + \beta \sigma [p_{i,0,1}V_{i,1}^* + p_{i,0,0}V_{i,0}^*] . \]
In that case, \( R_{L,1}(\bar{r}, q) = V_{L,1}^* \) in equation (32) can be written

\[
\begin{align*}
\varphi(\bar{r} + \alpha - y_L) & (1 - \beta(1 - \sigma)p_{L,0,0}) \\
= & -V_{L,1}^*[1 - \beta(1 - \sigma)][1 - \beta(1 - \sigma)\Delta p_L] + [1 - \beta(1 - \sigma)\Delta p_L] \cdot q\mathcal{E}(Q) \\
& + \beta(1 - \sigma)p_{L,1,0} \cdot \beta \sigma \mathbb{E}_0[V_{L}^*] + (1 - \beta(1 - \sigma)p_{L,0,0}) \beta \sigma \mathbb{E}_1[V_{L}^*]
\end{align*}
\]

(35)

Given \( q\mathcal{E}(Q) \) and \( V_{L,1}^* \) and denoting \( \hat{\beta} = \beta(1 - \sigma) \), we can solve equation (35) for \( \bar{r} \).

\[
\bar{r} = y_L - \alpha + \varphi^{-1} \left( \frac{(1 - \hat{\beta}\Delta p_L)(q\mathcal{E}(Q) - (1 - \hat{\beta})V_{L,1}^*) + \hat{\beta}p_{L,1,0} \cdot \beta \sigma \mathbb{E}_0[V_{L}^*] + \left( 1 - \hat{\beta}p_{L,0,0} \right) \beta \sigma \mathbb{E}_1[V_{L}^*]}{1 - \hat{\beta}p_{L,0,0}} \right)
\]

Here we can see that renters are willing to pay more if the flow value of housing is large (i.e., \( q\mathcal{E}(Q) \) is large), if the expected discounted utility from leaving and searching for a new rental is small (i.e. \( V_{L,1}^* \) is small), if the amount of extra work required is small (i.e. \( y_L - \alpha \) is larger), or if they expect to be in the unemployed state frequently (\( p_{L,0,0} \) is small).

### D.4 Equilibrium Evictions with Variable Rate Contracts

We now parameterize \( \varphi(h^+) = \frac{1}{\varphi_0} h^+ \) and use the extended model to make two points. First, we use our calibrated parameters to find a value of \( \varphi_0 \) so that our baseline equilibrium in the constant rent model is an equilibrium in the extended model with variable contracts, two-sided lack of commitment, and overtime. One can think of this calculation as consistent with a renegotiation of the fixed rate contract by a measure zero pair in our existing fixed rate equilibrium (and hence the parameters and endogenous values such as \( V_{L,e}, q_{L,e}, \) and \( Q \) apply). Second, we provide a sufficient condition for \( \bar{r} < r \), so that evictions occur in equilibrium in the extended model. Under this calculation, we allow for every equilibrium object to be endogenous to the model with variable rent contracts and derive a sufficient condition for evictions to occur by bounding \( V_{L,e}^* \) appropriately.

We start with our calibrated economy and plug in our calibrated parameters and equilibrium values of \( V_{L,e}, q_{L,e}, \) and \( Q \) to calculate \( \phi_0 \) such that

\[
\begin{align*}
\varphi_0 & \left( \frac{(1 - \hat{\beta}\Delta p_L)(q\mathcal{E}(Q) - (1 - \hat{\beta})V_{L,1}^*) + \hat{\beta}p_{L,1,0} \cdot \beta \sigma \mathbb{E}_0[V_{L}^*] + \left( 1 - \hat{\beta}p_{L,0,0} \right) \beta \sigma \mathbb{E}_1[V_{L}^*]}{1 - \hat{\beta}p_{L,0,0}} \right) \\
& + y_L - \alpha \leq \left( \frac{1 - \beta \Delta p_L}{p_{L,0,1}} \right) f_L
\end{align*}
\]

Here we can see that renters are willing to pay more if the flow value of housing is large (i.e., \( q\mathcal{E}(Q) \) is large), if the expected discounted utility from leaving and searching for a new rental is small (i.e. \( V_{L,1}^* \) is small), if the amount of extra work required is small (i.e. \( y_L - \alpha \) is larger), or if they expect to be in the unemployed state frequently (\( p_{L,0,0} \) is small).
We find that any value of $\phi_0 \leq 0.06$ is sufficient to strictly satisfy this inequality. For such a $\phi_0$, our calibrated model equilibrium without overtime and constant rent contracts remains an equilibrium in this extended model. To interpret this value, consider that landlords must raise rent by 0.3 units in order to keep an unemployed tenant. With $\phi_0 = 0.06$, the disutility of working overtime to pay that rent is equal to 5, which is 2.5 the $L-$type’s normal income.

Next we find a sufficient condition for evictions to occur in equilibrium with variable rate contracts. For simplicity, we consider the case of $\sigma = 0$. First, note that $r$ must be smaller than the value found by setting $V^*_{L,1} = 0$, $q = \bar{q}$, $Q = \mathcal{E}(\bar{q})$. That is,

$$r \leq \phi_0 \left( \frac{(1 - \beta \Delta p_L)\bar{q}\mathcal{E}(\bar{q})}{1 - \beta p_{L,0,0}} \right) + y_L - \alpha$$

(36)

Therefore, the interval $[\underline{r}, \bar{r}]$ is empty (and the $L-$type is evicted in equilibrium) if

$$\phi_0 \left( \frac{(1 - \beta \Delta p_L)\bar{q}\mathcal{E}(\bar{q})}{1 - \beta (1 - \sigma)} \right) + y_L - \alpha < \left( \frac{1 - \beta \Delta p_L}{p_{L,0,1}} \right) f_L,$$

which is true for sufficiently low $\phi_0$. If $\sigma > 0$ then we use the same logic to get a sufficient condition by noting that $E_{\epsilon}[V^*_{L}] \leq \frac{\mathcal{E}(\bar{q})}{1 - \beta (1 - \sigma)}$. Note that we have plugged in values to provide the most conservative possible upper bound on $\bar{r}$ (i.e. we assumed the current match was as valuable as possible whereas the value of walking away ($V^*_{L,1}$) was as small as possible). Therefore, larger values of $\phi_0$ (lower marginal disutilities of labor supply) will generally be consistent with evictions in equilibria of the model with overtime and two-sided lack of commitment.

**E Appendix: Aggregate Uncertainty**

A landlord who has a renter with constant rent $r$ and housing quality $q$ has the following values:

$$L_{i,1}(r, q; s) = r - f_i + \beta(1 - \sigma)\mathbb{E} \left[ p_{i,1,1}(s)L_{i,1}(r, q; s') + p_{i,1,0}(s)L_{i,0}(r, q; s') \right],$$

$$L_{i,0}(r, q; s) = \max_{\epsilon \in \{0, 1\}} -f_i + \beta(1 - \sigma)(1 - \epsilon)\mathbb{E} \left[ p_{i,0,1}(s)L_{i,1}(r, q; s') + p_{i,0,0}(s)L_{i,0}(r, q; s') \right].$$
A renter in a unit of quality $q$ with constant rent $r$ has the following values:

$$R_{i,1}(r, q; s, \mu) = y_i - \alpha + q\mathcal{E}(Q) - r + \beta(1-\sigma)\mathbb{E}\left[p_{i,1,1}(s)R_{i,1}(r, q'), s', \mu'\right] + p_{i,1,0}(s)R_{i,0}(r, q; s', \mu')$$

$$+ \beta\sigma \mathbb{E}\left[p_{i,1,1}(s)V^*_i(s', \mu') + p_{i,1,0}(s)V^*_i(s', \mu')\right]$$

$$R_{i,0}(r, q; s, \mu) = q\mathcal{E}(Q) + \beta\mathbb{E}\left[(1-\sigma)(1-\epsilon)(p_{i,0,1}(s)R_{i,1}(r, q; s', \mu') + p_{i,0,0}(s)R_{i,0}(r, q; s', \mu'))\right]$$

$$+ (1-(1-\sigma)(1-\epsilon))(p_{i,0,1}(s)V^*_i(s', \mu') + p_{i,0,0}(s)V^*_i(s', \mu')) \right].$$

F Appendix: Computation

Here we summarize the algorithm used to compute the different equilibria in this paper.

F.1 Computation of Stationary Equilibrium

We solve for the equilibrium numerically by discretizing the value function across grids, $(r, q) \in \mathcal{R} \times \mathcal{Q}$. We first numerically solve for the landlord’s value across $\mathcal{R} \times \mathcal{Q}$, which is independent of $\mathcal{Q}$. We can then invert the free entry conditions to yield equilibrium submarket tightnesses $\theta_{i,e}(r, q)$ which are also independent of $\mathcal{Q}$. We then guess $Q^n$, solve the conditional renter values and policies using value function iteration on $\mathcal{R} \times \mathcal{Q}$, compute the stationary measure by iterating on equations (10) and (11), and calculate $Q^{n+1}$ implied by the stationary measure. We repeat the process until convergence.

1. Discretize $(r, q) \in \mathcal{R} \times \mathcal{Q}$ for appropriate grids $\mathcal{R}, \mathcal{Q}$.

2. Compute $L_{i,e}(r, q)$ numerically, and invert the free entry condition to yield $\theta_{i,e}(r, q)$.

3. Initialize guesses: $Q^0, V^{*,0}_{i,e}, R^{0}_{i,e}$.

4. Given guesses $Q^n, V^{*,n}_{i,e}, R^{n}_{i,e}$ compute updates to $V^{*,n+1}_{i,e}, R^{n+1}_{i,e}$ and search policies by solving (6), (7), (8).

5. Given the search policies, iterate on (10) and (11) to solve for the stationary distribution $\mu_{i,e}^{h,n}(r, q)$. Compute $Q^{n+1}$ as implied by $\mu_{i,e}^{h,n}(r, q)$.

6. Check for convergence of all equilibrium objects. If not converged, return to 4.
F.2 Computation of Equilibrium with Aggregate Uncertainty

Due to the externality term $\mathcal{E}(Q)$, the version of the model with aggregate uncertainty as described in section 8 cannot be solved exactly and hence we approximate the solution in the spirit of Krusell and Smith (1998) [15]. In particular, while the true renter problem includes in the state space the entire distribution of renters, $\mu_{i,e}$, we approximate the solution by replacing the distribution of renters in the state space with a summary statistic - in our case, $Q$ itself. We guess a forecasting rule $Q' = a(s) + b(s)Q$, discretize $Q \in \tilde{Q}$, solve for the renter values and policies, simulate a pseudopanel of households using the policy rules, and update the forecasting rule $a(s), b(s)$ using forecasting regressions implied by our pseudopanel. We repeat until convergence of the forecasting rule. To summarize, we compute the equilibrium using the following steps:

1. Discretize $(r, q) \in R \times Q$ for appropriate grids $R, Q$.

2. Compute $L_{i,e}(r, q; s)$ numerically, and invert the free entry condition to yield $\theta_{i,e}(r, q; s)$.

3. Discretize $Q \in \tilde{Q}$, and initialize guesses of $a^0(s), b^0(s)$.

4. Given guesses $a^n(s), b^n(s)$ compute renter values and policies.

5. Given the search policies, simulate a pseudopanel of $N = 10000$ households for a sample of $T = 4500$ periods, in addition to a $T_0 = 500$ period burn-in.

6. Compute $a^{n+1,u}(s), b^{n+1,u}(s)$ from forecasting regressions.

7. Check for convergence of all equilibrium objects. If not converged, update $a^{n+1} = \rho a^{n+1,u} + (1 - \rho) a^n(s)$ and $b^{n+1} = \rho b^{n+1,u} + (1 - \rho) b^n(s)$ for tuning parameter $\rho \in (0, 1]$ and return to 4.

F.3 Appendix: Definition of Model Statistics

Here we define summary statistics reported in our tables and figures throughout the paper.

In describing Table 5 we mention duration statistics from our model. The average rental duration for $H$-types is simply $d_H = 1/\sigma$. The average rental duration for $L$-types is $d_L = \frac{1}{\mu_{H,1}^h + \mu_{H,0}^h + \mu_{L,1}^h + \mu_{L,0}^h}$. The total average rental duration is then

$$d = \frac{d_H(\mu_{H,1}^h + \mu_{H,0}^h) + d_L(\mu_{L,1}^h + \mu_{L,0}^h)}{\mu_{H,1}^h + \mu_{H,0}^h + \mu_{L,1}^h + \mu_{L,0}^h}.$$
In Table 8 we report newborn consumption equivalent (CE) values for each type. The calculation is simplified by the fact that agents have linear preferences. The type i newborn value is defined as $V_{i, nb} = \bar{p}_i V_{i, \lambda}^* + (1 - \bar{p}_i) V_{i, 0}^*$ where $\bar{p}_i$ is the long run probability of a type-i renter being employed as implied by the invariant distribution of type i’s employment Markov process. Then, the associated CE value which measures how much a newborn of type i would be willing to pay (if positive) or need to be paid (if negative) as a fraction of per period consumption to go from our baseline economy (where $\eta = 0$) to one where there are no spillovers (i.e. $\eta = 0$) is defined as:

$$CE_i = \frac{(V_{i, \lambda}^*(\eta = 0) - V_{i, nb}(\text{base}))}{V_{i, nb}(\text{base})}. \quad (37)$$

In Table 12 we calculate aggregate welfare differences associated with competitive equilibrium under different assumptions relative to the planner’s allocation. To do so, first we compute $W_{\text{sp}}^{\varepsilon(Q_{sp})}$ as the value of the social planner’s objective function (described in Appendix B in equation (14)) evaluated at the planner’s allocation $((q_{i,e}^{sp}, \theta_{i,e}^{sp}), \forall (i, e) \in \{H, L\} \times \{0, 1\})$ and the $\mu_{i,e}^j(\theta_{i,e}^{sp})$ implied by the planner’s allocation. Then we compute $W_{\text{base}}^{\varepsilon(Q_{sp})}$ as aggregate social surplus associated with our baseline decentralized equilibrium (i.e. the maximand from equation 14 evaluated at the decentralized values of $((q_{i,e}, \theta_{i,e}), \forall (i, e) \in \{H, L\} \times \{0, 1\}$ and $\mu_{i,e}^j$ implied by the competitive equilibrium allocation and eviction policies). Finally, we compute $W_{\text{base}}^{\varepsilon(Q_{sp})}$ as the objective function obtained by the decentralized equilibrium, if the externality term was evaluated at the externality value associated with the planner’s allocation. The quantities reported in the table are then:

$$\text{Baseline } Q = \frac{W_{\text{base}}^{\varepsilon(Q_{sp})} - W_{\text{sp}}^{\varepsilon(Q_{sp})}}{W_{\text{sp}}^{\varepsilon(Q_{sp})}}, \quad (38)$$

$$\text{Planner } Q = \frac{W_{\text{base}}^{\varepsilon(Q_{sp})} - W_{\text{sp}}^{\varepsilon(Q_{sp})}}{W_{\text{sp}}^{\varepsilon(Q_{sp})}}. \quad (39)$$

In Figure 2 we report the CE value for L-type renters in equilibria characterized by alternative moratorium policies. Specifically, we calculate $CE_L = \frac{V_{L, 1}^*(\lambda) - V_{L, 1}^*(\lambda = 1)}{V_{L, 1}^*(\lambda = 1)}$ for $\lambda \in [0, 1]$. We report the same quantities in Figure 4 where we additionally vary $\alpha \in \{1, 0.9\}$. Comparisons are within-$\alpha$, i.e. for this figure we report:

$$CE_L = \frac{V_{L, 1}^*(\lambda, \alpha) - V_{L, 1}^*(\lambda = 1, \alpha)}{V_{L, 1}^*(\lambda = 1, \alpha)}, \quad (40)$$

for each $\alpha$.

Figure 5 reports type-specific newborn welfare calculations expressed in terms of CE along
with a non-type specific newborn welfare relative to the no-subsidy decentralized equilibrium. Specifically, the type \( i \)-specific \( \text{CE}_i \) numbers refer to:

\[
\text{CE}_i = \frac{V_i^{nb}(S) - V_i^{nb}(S = 0)}{V_i^{nb}(S = 0)}. \tag{41}
\]

The non-type-specific \( \text{CE} \) numbers further integrate out the type of household, i.e. \( V^{nb} = \sum_i \mu_i V_i^{nb} \) and \( \text{CE} \) refers to:

\[
\text{CE} = \frac{V^{nb}(S) - V^{nb}(S = 0)}{V^{nb}(S = 0)}. \tag{42}
\]

Finally, we report \( \text{CE} \) numbers for the aggregate uncertainty case in Table 15 relative to our baseline aggregate economy without eviction restrictions. We average the values of each type under each \( \lambda(s) \) policy and then compute the welfare metric. That is, let \( \bar{V}_i(\lambda(G), \lambda(B)) \) be the average value of a renter of type \( i \) under a given set of policies, then \( \text{CE}_i \) is:

\[
\text{CE}_i = \frac{\bar{V}_i(\lambda(G), \lambda(B)) - \bar{V}_i(1,1)}{\bar{V}_i(1,1)}. \tag{43}
\]

\section*{G Appendix: Calculation of \( \Lambda^* \) Matrix}

We first compute the \( \Lambda \) matrix from Andrews, Gentzkow, and Shapiro (2017) [3]. The matrix is defined as:

\[
\Lambda = - (G'WG)^{-1} G'W \tag{44}
\]

where \( G = \mathbb{E} [\nabla \bar{g}(b)] \) is the 5 \( \times \) 4 probability limit of the Jacobian and \( W \) is the probability limit of the weighting matrix, which we have simply taken to be the identity matrix. \( \Lambda \) measures how sensitive the parameter estimates are to local perturbations of the data moments. Further, there is a tight connection between \( \Lambda \) and standard errors in GMM/SMM. Specifically, given (44), the limiting distribution of the estimates can be written

\[
\sqrt{T} \left( \hat{b} - b_0 \right) \xrightarrow{d} \mathcal{N} [0, \Lambda \Omega \Lambda'] \tag{45}
\]

where \( \Omega = \mathbb{E} [g(b)g(b)'] \) is the limiting variance-covariance matrix of the data moments, \( b_0 \) is the true parameter value, and \( T \) is sample size. For a given \( \Omega \), (45) makes clear that small values of \( \Lambda \) are associated with more precise parameter estimates.
Table 18: AGS Sensitivity Matrix $\Lambda$

<table>
<thead>
<tr>
<th></th>
<th>$r_H/y_H$</th>
<th>eviction rate</th>
<th>r/q slope</th>
<th>match elasticity</th>
<th>spillover</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
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<td>-1.31</td>
<td>-1.98</td>
<td>-0.12</td>
</tr>
<tr>
<td>$\nu$</td>
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<td>-2.66</td>
<td>5.04</td>
<td>8.49</td>
<td>0.07</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>-0.07</td>
<td>-0.01</td>
<td>0.1</td>
<td>0.15</td>
<td>-0.05</td>
</tr>
<tr>
<td>$c_0$</td>
<td>69.71</td>
<td>5.99</td>
<td>-59.6</td>
<td>-82.05</td>
<td>-4.74</td>
</tr>
</tbody>
</table>

Once we have computed $\Lambda$, we then rescale each element of $\Lambda$ to calculate $\Lambda^*$ as is reported in Table 7. Specifically, while the $(i, j)$'th element of $\Lambda$ corresponds to $\frac{\partial b_i}{\partial m_j}$ where $m_j$ is the j’th moment, we define $\Lambda^*$ such that the $(i, j)$’th element refers to $\frac{\partial b_i}{\partial m_j} \times m_j b_i$. $\Lambda^*$ hence is a matrix of elasticities of the parameter estimates with respect to the data moments, as opposed to the derivative of the parameter estimates with respect to the data moments. We note that the derivative depends on the scaling of the parameters and moments whereas the elasticity is invariant to the scale of these objects. Hence, we report $\Lambda^*$ in Table 7.

In practice, we approximate $G$ with a finite-difference approximation. That is, when computing the derivative of moment $j$ with respect to parameter $i$, we approximate $\frac{\partial \hat{g}_j(b)}{\partial b_i} \approx \frac{\hat{g}_j(b+s_j) - \hat{g}_j(b)}{s_j}$, where $s_j$ is the step size chosen for parameter $j$ and $\vec{s}_j$ is a $4 \times 1$ vector containing $s_j$ as the $j$-th element and 0 for all other elements. We use a 5 percent finite-difference so $s_j = 0.05 \times \theta_j$.

To extend our discussion of identification within our model beyond the 4 that we target to match moments, we compute an extended Jacobian where the set of moments considered includes additional equilibrium policies and quantities and our set of parameters includes the parameters that we calibrate outside of the model. We report this extended Jacobian in Appendix H.

**H Appendix: Jacobian**
### Table 19: Expanded Jacobian

<table>
<thead>
<tr>
<th></th>
<th>$\xi$</th>
<th>$\nu$</th>
<th>$C_0$</th>
<th>$\kappa$</th>
<th>$p_{L,1}$</th>
<th>$p_{H,1}$</th>
<th>$p_{L,0}$</th>
<th>$p_{H,0}$</th>
<th>$f_L$</th>
<th>$f_H$</th>
<th>$y_L$</th>
<th>$y_H$</th>
<th>$\mu_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{H}/y_H$</td>
<td>1.36</td>
<td>-0.03</td>
<td>-0.03</td>
<td>0.12</td>
<td>0.0</td>
<td>-0.42</td>
<td>0.0</td>
<td>-0.04</td>
<td>0.0</td>
<td>0.33</td>
<td>0.0</td>
<td>-0.11</td>
<td>0.0</td>
</tr>
<tr>
<td>Eviction rate</td>
<td>3.69</td>
<td>0.41</td>
<td>-0.05</td>
<td>-2.92</td>
<td>3.29</td>
<td>0.1</td>
<td>2.18</td>
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<td>-1.62</td>
<td>0.22</td>
<td>0.56</td>
<td>0.0</td>
<td>4.17</td>
</tr>
<tr>
<td>$r/q$ slope</td>
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<td>-0.0</td>
<td>-0.19</td>
<td>0.83</td>
<td>-0.38</td>
<td>0.0</td>
<td>-0.02</td>
<td>0.06</td>
<td>0.06</td>
<td>-0.04</td>
<td>0.0</td>
<td>0.05</td>
</tr>
<tr>
<td>Experiment</td>
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<td>0.57</td>
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<td>14.0</td>
<td>7.84</td>
<td>43.43</td>
<td>8.31</td>
<td>19.74</td>
<td>11.99</td>
<td>3.91</td>
<td>1.87</td>
<td>73.01</td>
</tr>
<tr>
<td>Vacancy rate</td>
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<td>-95.79</td>
<td>-10.48</td>
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<td>-148.74</td>
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</tr>
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<td>$q_{L,1}$</td>
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<td>-0.0</td>
<td>-0.3</td>
<td>1.04</td>
<td>-0.01</td>
<td>0.0</td>
<td>0.0</td>
<td>0.08</td>
<td>-0.01</td>
<td>0.41</td>
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<tr>
<td>$q_{H,1}$</td>
<td>3.91</td>
<td>-0.07</td>
<td>-0.16</td>
<td>0.31</td>
<td>0.0</td>
<td>0.2</td>
<td>0.0</td>
<td>-0.05</td>
<td>0.0</td>
<td>1.19</td>
<td>0.0</td>
<td>0.0</td>
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<tr>
<td>$q_{H,0}$</td>
<td>3.93</td>
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<td>-0.16</td>
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</tr>
<tr>
<td>$Q$</td>
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<td>0.01</td>
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<tr>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
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<td>1.0</td>
<td>0.0</td>
<td>0.0</td>
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</tr>
<tr>
<td>$r_{H,1}$</td>
<td>5.43</td>
<td>-0.01</td>
<td>-0.14</td>
<td>0.49</td>
<td>0.0</td>
<td>-1.69</td>
<td>0.0</td>
<td>-0.16</td>
<td>0.0</td>
<td>1.29</td>
<td>0.0</td>
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<td>0.0</td>
</tr>
<tr>
<td>$r_{H,0}$</td>
<td>5.54</td>
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<td>-0.14</td>
<td>0.49</td>
<td>0.0</td>
<td>-1.84</td>
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<tr>
<td>$\theta_{L,1}(r_{L,1},q_{L,1})$</td>
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<td>33.1</td>
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<td>0.0</td>
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<td>-5.27</td>
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<td>10.72</td>
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<tr>
<td>$\theta_{H,1}(r_{H,1},q_{H,1})$</td>
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<td>0.38</td>
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</tr>
<tr>
<td>$\theta_{H,0}(r_{H,0},q_{H,0})$</td>
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<td>0.0</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
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</tr>
<tr>
<td>$\phi(\theta_{L,1}(r_{L,1},q_{L,1}))$</td>
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<td>0.27</td>
<td>-0.03</td>
<td>-1.83</td>
<td>2.07</td>
<td>0.07</td>
<td>0.0</td>
<td>0.0</td>
<td>-1.02</td>
<td>0.14</td>
<td>0.37</td>
<td>0.0</td>
<td>-0.65</td>
</tr>
<tr>
<td>$\phi(\theta_{H,1}(r_{H,1},q_{H,1}))$</td>
<td>0.03</td>
<td>0.12</td>
<td>-0.01</td>
<td>-0.16</td>
<td>0.0</td>
<td>0.03</td>
<td>0.0</td>
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<td>-0.23</td>
<td>0.0</td>
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<td>0.0</td>
</tr>
<tr>
<td>$\phi(\theta_{H,0}(r_{H,0},q_{H,0}))$</td>
<td>0.99</td>
<td>0.13</td>
<td>-0.0</td>
<td>-0.18</td>
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</tbody>
</table>

Note: Boxed upper-left corner is Jacobian matrix from the computation of $A$ from Andrews, Gentzkow, and Shapiro (2017) [3].