

Negative Nominal Interest Rates Can Worsen Liquidity Traps *

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September 10, 2019

Abstract

Negative nominal interest rates are contractionary in an expectations-driven liquidity trap, unless they are able to eliminate deflationary expectations altogether. If firms expect low demand, then they may find it optimal to reduce prices, leading an active central bank to reduce nominal rates to the (possibly negative) lower bound, raising real rates and fulfilling the pessimistic expectations of price setters. Locally, a further reduction in the nominal rate is met by even more deflation. Globally, there exists a sufficiently negative nominal rate that eliminates liquidity trap equilibria, as long as monopoly power and price adjustment costs place some limit on firms' willingness to reduce prices in response to competitors' deflation. Quantitatively, the nominal rate required to eliminate liquidity traps is typically below -100% for realistic parameters.

Preliminary

*I thank David Andolfatto, Huixin Bi, Jose Mustre-Del-Rio, and Andre Kurmann for helpful discussions about this paper. Any errors are my own. The views expressed in this paper are those of the author and do not necessarily represent the views of the Federal Reserve Bank of Kansas City or the Federal Reserve System.

1 Introduction

Should central banks attempt to set negative nominal interest rates in order to overcome the adverse effects of the zero lower bound? This note provides one point of caution against such a policy. I work with the sticky-price New Keynesian model with two steady-state equilibria, one that is “intended” with high output and inflation and one that has low output and deflation, representing an expectations-driven liquidity trap ala Benhabib, Schmitt-Grohe, and Uribe [1],[2].¹ Locally, the liquidity trap equilibrium is “neo-fisherian”: reducing the nominal interest rate is contractionary in such an equilibrium. Reducing the nominal rate below zero simply allows for price-setters to conceive of more deflation, to which they respond by cutting their prices more severely than when the nominal rate was zero.

While the negative rate is locally contractionary, it may be expansionary if sufficiently bold. There is a limit on how quickly firms are willing to cut prices, since it is costly to do so and the benefits are limited by the price elasticity of goods demand. Therefore, if rates can be set sufficiently negative that expected deflation is greater than this limit, then a negative interest rate policy can eliminate the liquidity trap equilibrium and stabilize the economy. However, a calibrated version of the model requires negative rates that are much larger than previously discussed. In fact, it is below -100% for reasonable parameterizations of the model.

This paper is related to the growing literature on negative nominal rate policies. While many papers focus on the institutional arrangements necessary to actually achieve negative nominal rates, I take the ability of central banks to set negative rates as given and focus on the effects of doing so in a workhorse macroeconomic model. Similarly, Buitert and Panigirtzoglou [3] study a New Keynesian model with liquidity traps equilibria and the possibility of negative nominal rates. They consider the effects of this policy on inflation in an endowment economy and find that the negative interest rate policy *must* eliminate the liquidity trap equilibrium, whereas it may actually make it worse when output is endogenous. Rognlie [15] also studies the effect of negative interest rates in a New Keynesian model, but he assumes that prices are completely rigid, which corresponds to a version of the present model in which the expectations-driven liquidity trap does not exist in the first place because price adjustment costs are infinite. Finally, Eggertsson, Juelsrud, and Wold [7] and Eggertsson, et al. [6] consider new models in which negative nom-

¹There is also a continuum of non-stationary equilibria that converge to the liquidity trap steady state. I will focus on the steady state, since eliminating it will ensure that the intended equilibrium is unique.

inal rates are contractionary through an alternative mechanism that involves banks passing their losses from holding negative yielding debt to customers as higher interest rates.

Another set of papers do not consider NIRP explicitly, but highlight the Neo-Fisherian effects of nominal rates in liquidity trap equilibria. Schmitt-Grohe and Uribe [16] show that an interest rate rule that jumps to a higher nominal peg during the liquidity trap is expansionary. A similar result is shown by Cuba-Borda and Singh [4] in a model with both expectations-driven liquidity traps and secular stagnation. To my knowledge, this is the first paper to highlight the non-monotone effect of negative nominal rates in the liquidity trap by calculating the threshold for which only the intended equilibrium exists.

I will now set up the model and solve for steady-states in section 2. Qualitative comparative statics are done in section 3, while section 4 uses estimates of parameter values to calculate the required NIRP to eliminate liquidity traps. Section 5 then asks what alterations to the model would allow small negative interest rates to be expansionary, either by changing the local comparative statics or by eliminating the liquidity trap more quickly. I then conclude.

2 Model

The model features Rotemberg price adjustment costs in continuous time. The economy comprises a continuum of consumer-producer households, each of which sells a differentiated good indexed by j . The household chooses sequences,

$$\{(c_{\ell,t})_{\ell \in [0,1]}, n_t, \pi_t, p_{j,t}, N_t\}_{t=0}^{\infty}$$

to maximize:

$$\int_0^{\infty} e^{-\delta t} \left[\log \left[\int_0^1 c_{\ell,t}^{\frac{\epsilon-1}{\epsilon}} d\ell \right]^{\frac{\epsilon}{\epsilon-1}} - \Psi \frac{\nu}{1+\nu} n_t^{\frac{1+\nu}{\nu}} - 0.5\gamma\pi_t^2 \right] dt, \quad (1)$$

subject to the constraints

$$\dot{b}_t + \int_0^1 p_{\ell,t} c_{\ell,t} d\ell = i_t b_t + W_t n_t + Profit_{j,t} \quad (2)$$

$$Profit_{j,t} = p_{j,t} y_{j,t}^d(p_{j,t}) - W_t N_t \quad (3)$$

$$\dot{p}_{j,t} = \pi_t p_{j,t} \quad (4)$$

$$N_t \geq y_{j,t}^d(p_{j,t}) \quad (5)$$

where the nominal wages and interest rates are taken as given.

The wage is determined by labor market clearing, while the nominal interest rate is set by policy according to a Taylor Rule with two key features. First, the nominal rate responds more than one for one to changes to inflation. Second, the nominal rate is bound below by $-\zeta$. That is,

$$i_t = \max\{\delta + \phi\pi_t, -\zeta\}, \quad (6)$$

with $\phi > 1$ and $\zeta \geq 0$.

2.1 Equilibrium System

Two equations govern the dynamics of output and inflation:

$$\frac{\dot{Y}_t}{Y_t} = \max\{(\phi - 1)\pi_t, -\zeta - \delta - \pi_t\}, \quad (7)$$

$$\dot{\pi}_t = \delta\pi_t - \frac{\epsilon}{\gamma}\Psi Y_t^{\frac{1+\nu}{\nu}} + \frac{\epsilon - 1}{\gamma}. \quad (8)$$

2.2 Steady State Equilibria

There is always at least one steady-state equilibrium, which is labelled as the “intended” equilibrium with $\pi^I = 0$ and $y^I = \left(\frac{\epsilon - 1}{\Psi\epsilon}\right)^{\frac{\nu}{1+\nu}}$. However, there may also be an expectations-driven liquidity trap steady-state in which the nominal rate is equal to $-\zeta$ and inflation and output are given by

$$\pi^Z = -(\zeta + \delta), \quad (9)$$

$$Y^Z = \left[\frac{\epsilon - 1}{\Psi\epsilon} - \frac{\delta\gamma(\zeta + \delta)}{\Psi\epsilon} \right]^{\frac{\nu}{1+\nu}}. \quad (10)$$

This steady state apparently exists under the condition that the steady-state Phillips Curve intercepts below $-(\zeta + \delta)$. I will assume that this is true throughout, which requires the following assumption on parameters:

$$-\frac{\epsilon - 1}{\gamma\delta} < -(\zeta + \delta). \quad (11)$$

3 Comparative Statics and Discussion

I consider the effects of introducing a negative nominal rate in Figure (1). The blue, dashed curve corresponds to combinations of output and inflation

such that inflation is constant while the dotted red lines correspond to constant output and depend on ζ . Starting with $\zeta = 0$, the model has two steady-states, labelled (y^I, π^I) and $(y_{\zeta=0}^Z, \pi_{\zeta=0}^Z)$, the latter corresponding to the expectations-driven liquidity trap. Note the intercept of the Phillips Curve is drawn strictly below $-\delta$, which means that setting ζ to a small positive number shifts the steady-state Euler Equation previously associated with $\pi = -\delta$ to $\pi = -(\delta + \zeta^-)$, which now intersects the Phillips Curve at point $(y_{\zeta^-}^Z, \pi_{\zeta^-}^Z)$ (with no effect on the intended equilibrium, so $I_2 = I_1$). Therefore, a negative interest rate policy makes the liquidity trap equilibrium worse, at least locally.

Now consider what happens with a substantially negative nominal rate by setting ζ to

$$\zeta^+ > \zeta^* \geq \frac{\epsilon - 1 - \gamma\delta^2}{\gamma\delta}, \quad (12)$$

which is illustrated with the third and lowest steady-state Euler Equation. This is below the intercept of the Phillips Curve, which is at $-\frac{\epsilon-1}{\gamma}$. The two parameters, ϵ and γ , govern how negative nominal rates must be in order to eliminate the liquidity trap and do so in an intuitive way. The cost of changing prices is governed by γ . The larger it is the less deflation can be consistent with profit maximization and therefore the easier it is to eliminate the deflationary liquidity trap equilibrium. The second is the elasticity of substitution between goods, ϵ , which governs the marginal benefit of cutting prices when everybody else is expected to do so. The larger is this parameter the more sensitive is demand to the producer's relative price and the stronger her incentive to deflate when she expects others to do the same.

4 Quantitative Evaluation

It is useful to set some parameter values in order to ask how negative the nominal bound would need to be in order to eliminate the liquidity trap, as well as the effects of negative interest rates that are not large enough to do so.

4.1 Baseline Calibration

The model frequency is quarterly and $\delta = 0.002$, corresponding to a 0.8% annual real rate in the intended equilibrium. This is consistent with recent estimates of the natural rate using the models of Laubach and Williams [11] and Holston, Laubach, and Williams [10], but is much lower than historical

averages, which I will discuss below.² The Frisch elasticity of labor supply is set to $\nu = 0.75$ in the baseline calibration, while the disutility parameter Ψ is set to normalize output in the intended steady-state to one.

Economically, the adjustment cost and elasticity of substitution parameters are pivotal for the limits of deflation in the liquidity trap, yet they enter jointly in determining the intercept of the constant inflation locus. My strategy for choosing $\frac{\epsilon-1}{\gamma}$ is to assume that historical estimates of the Phillips Curve have used data near the intended equilibrium. As presented in Glover [9], the stochastic discrete-time version of this model would have the following Phillips Curve, after it is log-linearized around the intended steady-state:³

$$\pi_t = e^{-\delta} \mathbb{E}_t \pi_{t+1} + \left(\frac{\epsilon}{\gamma} \right) \left(\frac{1+\nu}{\nu} \right) \Psi (y^I)^{\frac{1+\nu}{\nu}} \left(\log y_t - \log y^I \right). \quad (13)$$

Assuming that labor supply is chosen to equate the marginal rate of substitution between consumption and labor to the real wage gives

$$\Psi (y^I)^{\frac{1+\nu}{\nu}} = \frac{w}{P} = \frac{\epsilon - 1}{\epsilon}, \quad (14)$$

which means that the theoretical slope of the Phillips Curve is $\frac{1+\nu}{\nu} \times \frac{\epsilon-1}{\gamma}$.

Mavroeidis, Plagborg-Moller, and Stock [12] review estimates of the slope of the Phillips Curve and report a range of 0.005 to 0.08 and their own estimate of 0.018. Using the low-point of their range of estimates in Equation 12 gives $\zeta^* = \frac{0.005}{0.002} \times \frac{0.75}{1.75} - 0.002 = 107\%$. Eliminating the liquidity trap equilibrium requires the lower bound on nominal rates to be well over -100% per quarter.

For values of ζ that are not sufficiently large to eliminate the liquidity trap, output and inflation fall monotonically. Figure 2 shows liquidity trap output and inflation as a function of the lower bound on nominal rates, using the above parameters. The decline in inflation is one-for-one, while the decline in output is concave, so that a small negative interest rate has little effect on output, but a large one can make the liquidity trap recession much more severe.

4.2 The Slope of the Phillips Curve

The steeper is the Phillips Curve in output, the more negative the nominal rate must be in order to eliminate the liquidity trap. The logic is that a

²The Federal Reserve of New York maintains up-to-date estimates of the natural rate using these models at the address <https://www.newyorkfed.org/research/policy/rstar>.

³The model in Glover [9] has heterogeneous labor supply and lower bounds on real wages, but gives the Phillips Curve in equation 13 if parameters are set so that only one labor type enters production and the real minimum wage is zero.

steeper Phillips Curve in this model must be due to either a larger elasticity of substitution or a smaller adjustment cost parameter. The former makes price-setters more responsive to the deflation of other goods and the latter simply makes it cheaper to respond. As shown in Figure 3, increasing the slope requires a more and more negative rate in order to eliminate the liquidity trap. The plot begins with a slope of 0.00023, which is the flattest Phillips Curve for which the liquidity trap exists. The upper bound is a slope of 0.024, for which the nominal rate required to eliminate the liquidity trap reaches minus 100 percent. Since 0.024 is well within the range of estimates provided by Mavroeidis, Plagborg-Moller, and Stock [12], there is a distinct possibility that the liquidity trap cannot be avoided through any negative interest rate policy.

4.3 The Natural Rate of Interest

The baseline calibration used a natural rate of 0.2% per quarter, which is consistent with recent estimates of the natural rate, but substantially lower than historical averages. How does a negative lower bound on nominal rates interact with a low natural rate? In the context of this model, a lower natural rate requires an increasingly negative nominal interest rate in order to eliminate the liquidity trap. Figure 4 plots the relationship of ζ^* as δ varies from 0.001 to 0.01, fixing the slope of the Phillips Curve at 0.005. The convexity of this curve highlights the heightened difficulty of eliminating the liquidity trap through negative nominal rates in a world of low natural rates.

4.4 Labor Cost of Price Adjustment

Thus far, I have chosen γ as a parameter. Another possibility is that price adjustments require labor, which must be paid the prevailing wage. This creates a state-contingent cost of price adjustment, i.e.

$$\gamma_t = \Psi Y_t^{\frac{1+\nu}{\nu}}. \quad (15)$$

This change in interpretation of adjustment costs fundamentally changes the ability of negative interest rates to eliminate liquidity traps, since the Phillips Curve is now given by

$$\dot{\pi}_t = \delta \pi_t - \epsilon + \epsilon Y_t^{-\frac{1+\nu}{\nu}}. \quad (16)$$

The constant-inflation locus is now drawn in Figure 5 as a concave curve that asymptotes to $-\infty$ as $y \rightarrow 0$. There is no longer a sufficiently negative lower bound on nominal rates to eliminate the self-fulfilling expectations of

deflation, since the cost of price adjustment endogenously declines with output, via wages.

5 When Is a Small NIRP Expansionary?

The model is simple enough to highlight two ways that a smaller negative interest rate could eliminate the liquidity trap. One possibility is to disconnect the intercept of the steady-state Phillips Curve from the estimated slope, such that the intercept is higher on the inflation axis, thereby requiring a smaller downward shift in the IS curve to eliminate the liquidity trap. The other is to fundamentally change the IS curve so that NIRP is locally expansionary in the liquidity trap equilibrium.

5.1 Wages Rigidities

In the basic model, firms who expect competitors to deflate due to low demand find it optimal to do so, partly because they expect their real wage bill to be low in such a setting. This is because lower output requires less labor, and therefore real wages are low in equilibrium. The presence of wage rigidities tends to flatten the Phillips Curve and affect the intercept (some studies that discuss this include Daly and Hobijn [5], Schmitt-Grohe and Uribe [16], and Glover [8], [9]). I now consider a version of the model with a hard lower bound on the real wage, based on the average real minimum wage in the U.S.⁴

Following Glover [9], the model with a real minimum wage of ω gives rise to the following Phillips Curve

$$\dot{\pi}_t = \delta\pi_t - \frac{\epsilon}{\gamma} \max \left\{ \omega, \Psi Y_t^{\frac{1+\nu}{\nu}} \right\} + \frac{\epsilon - 1}{\gamma}. \quad (17)$$

The resulting zero-inflation locus is drawn in Figure 6, in which there are two changes of note. First, the intercept has been raised by $\frac{\epsilon}{\gamma}\omega$. Second, the curve has become flat for output below

$$\underline{Y} = \left(\frac{\omega}{\Psi} \right)^{\frac{\nu}{1+\nu}}. \quad (18)$$

Quantitatively, ω can be calibrated using the minimum wage relative to average wages in the United States. The OECD reports this ratio to be 25%

⁴I model a real wage floor rather than a nominal. A nominal wage floor would itself eliminate the deflationary liquidity trap equilibrium, regardless of whether a NIRP was employed.

in 2017 [14] and the model gives a real wage in the intended equilibrium of $\frac{\epsilon-1}{\epsilon}$. I therefore set $\omega = 0.25\left(\frac{\epsilon-1}{\epsilon}\right)$. Using $\kappa = 0.005$, $\nu = 0.75$, and $\epsilon = 10$ gives $\gamma = 4200$. The new NIRP required to eliminate the liquidity trap is therefore

$$\zeta^* = \frac{10 - 1}{0.002 \times 4200} - \frac{10}{0.002 \times 4200} \times 0.25 \times \frac{9}{10} - 0.02 = 80.2\%. \quad (19)$$

Therefore, accounting for the effect of minimum wages on the intercept of the Phillips Curve suggests that a negative lower bound on interest rates of 80.2% per quarter is enough to eliminate the liquidity trap. Of course, this is still much more negative than what we have seen in reality and would be even more negative if we used lower natural rates, but demonstrates that wage rigidities make NIRP more effective.⁵

5.2 Wealth in the Utility

Michaillat and Saez [13] enrich the goods block of the New Keynesian model so that the constant-output locus is upward sloping in output when interest rates are at their lower bound. Furthermore, they calibrate the model so that the intercept of the liquidity trap IS curve is below the intercept of the Phillips Curve and the slope is steeper. Their differential equation for inflation is unchanged, but their expression for output growth is now

$$\frac{\dot{Y}_t}{Y_t} = \max\{(\phi - 1)\pi_t, -(\zeta + \delta) + \mu Y_t - \pi_t\}, \quad (20)$$

where $\mu > 0$ is a parameter that arises from households directly valuing wealth in their utility functions.

There are now two ways that a liquidity trap equilibrium could occur. One possibility is that δ and μ are both small, which gives a liquidity trap that is essentially the same as in the basic model. Another has a large value of both δ and μ , so that the liquidity trap intersection occurs with the constant-output locus cutting the constant-inflation locus from below, as shown in Figure 7. This is the case considered by Michaillat and Saez, who calibrate values of δ and μ to match laboratory estimates of time preference and the natural rate of interest. That is, they set $\delta = 0.43$ on an annual basis and target a natural rate of 0.02. Essentially, their calibration says that people are much

⁵This is an extreme example where the zero-inflation locus is unchanged above \underline{y} , but completely flat below. If the wage rigidity flattened the curve in addition to increasing the intercept, then there would be an additional output loss for values of ζ that are insufficient to eliminate the liquidity trap.

less patient regarding consumption than is typically assumed, but that they save nonetheless because they directly value the levels of their wealth.

In the Michaillat and Saez parameterization, there is no longer a neo-fisherian effect of negative nominal rates in the liquidity trap, nor is the effect non-monotone. Reducing the nominal rate to a negative value always pushes the IS curve downward, without changing the slope, and is therefore expansionary. In fact, the liquidity trap can occur at an intersection with positive inflation and higher than intended output, if a sufficiently negative rate is set.

Therefore, setting a negative lower bound on nominal rates may be globally expansionary, if the IS curve is shaped by wealth-in-the utility and if the natural rate largely reflects direct utility over wealth by households who heavily discount future consumption.

6 Conclusion

Negative nominal interest rates were thought impossible until very recently. The fact that countries now issue debt with negative yields has raised the question of whether a negative interest rate should be part of the monetary policy toolbox. While there is substantial uncertainty about how low negative rates could be set, it is a moot point if doing so is unlikely to stabilize the economy in the first place. This paper has shown that negative interest rates are likely to make things worse in an economy plagued by liquidity traps, given plausible estimates of the Phillips Curve and natural rate of interest, although there are some combinations of economic forces that could allow small negative rates to be expansionary.

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Figure 1: Effects of Varying Negative Nominal Rate

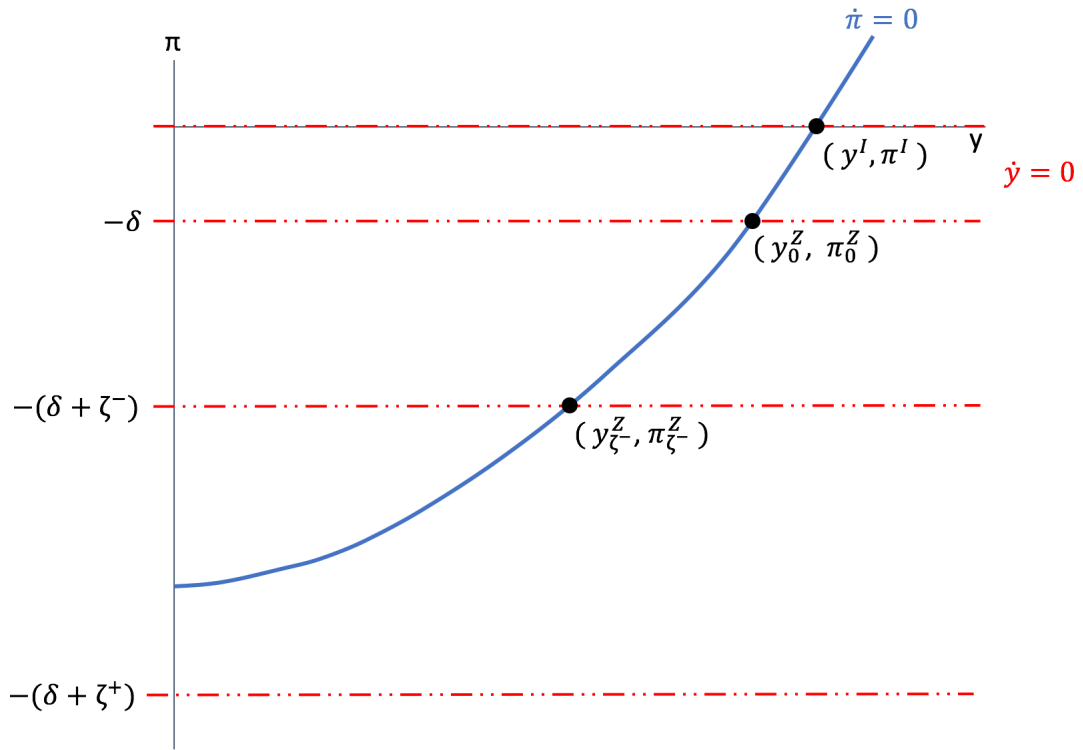


Figure 2: Liquidity Trap Equilibria Vary With NIRP

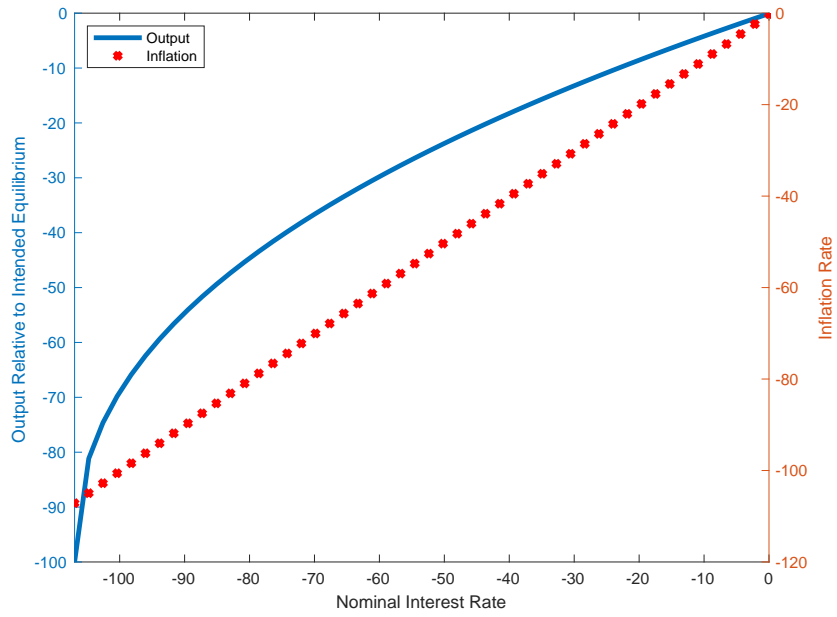


Figure 3: Required NIRP For Different Phillips Curve Slopes

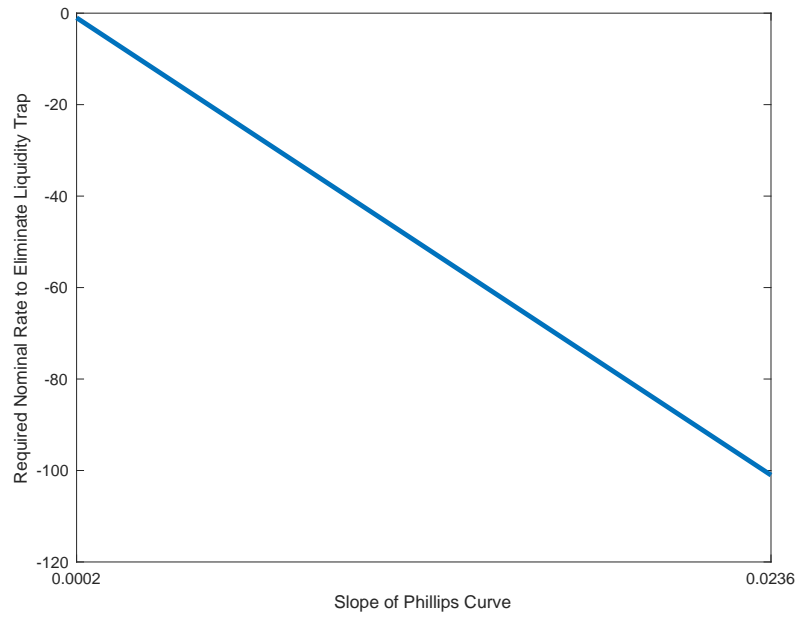


Figure 4: Required NIRP For Different Natural Rates

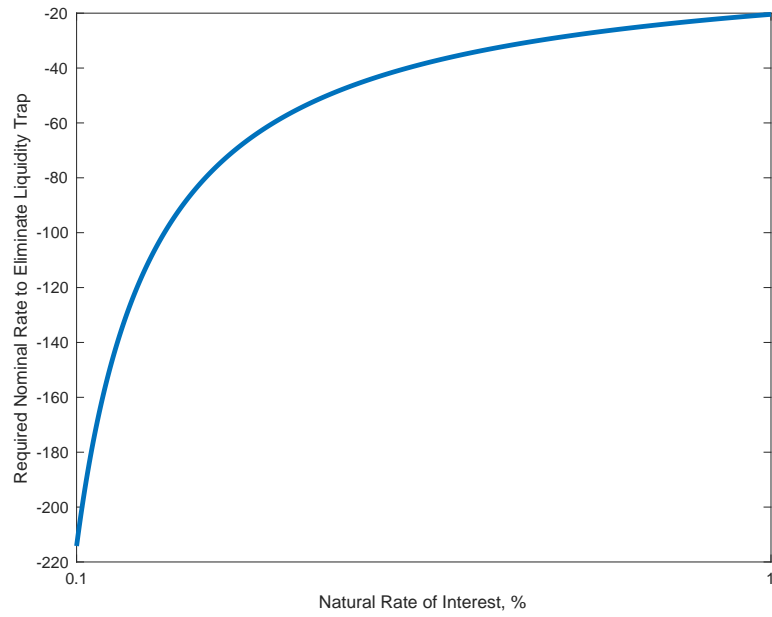


Figure 5: NIRP When Labor Used For Price Adjustment

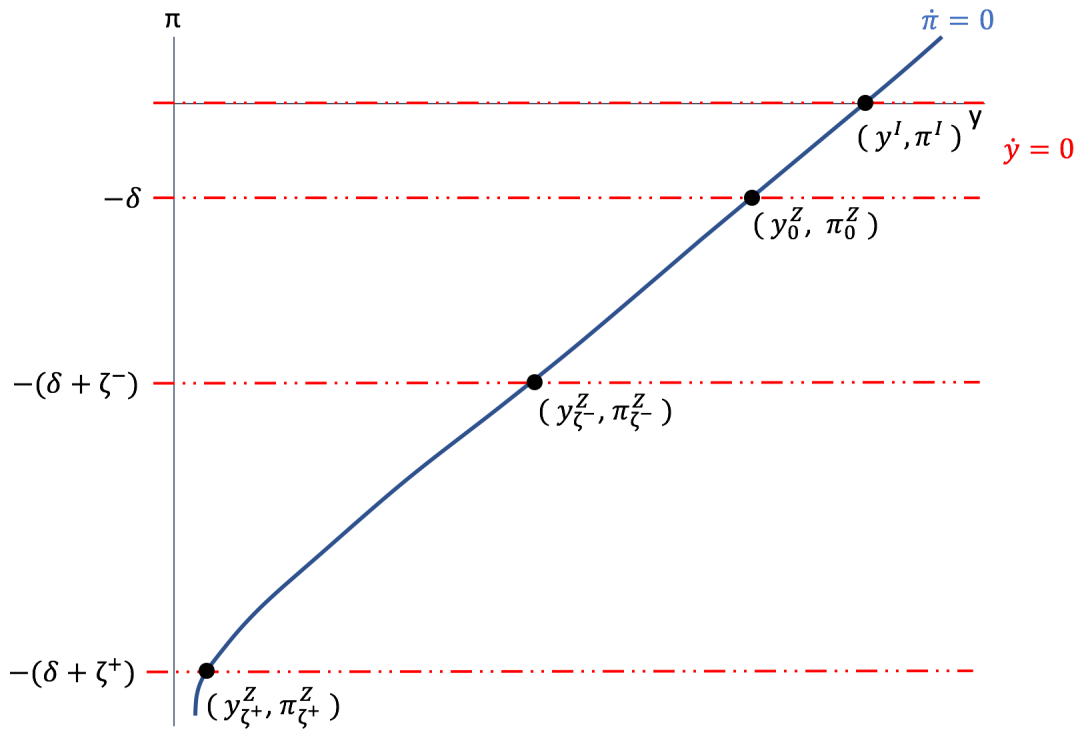


Figure 6: NIRP In Model With Minimum Wage

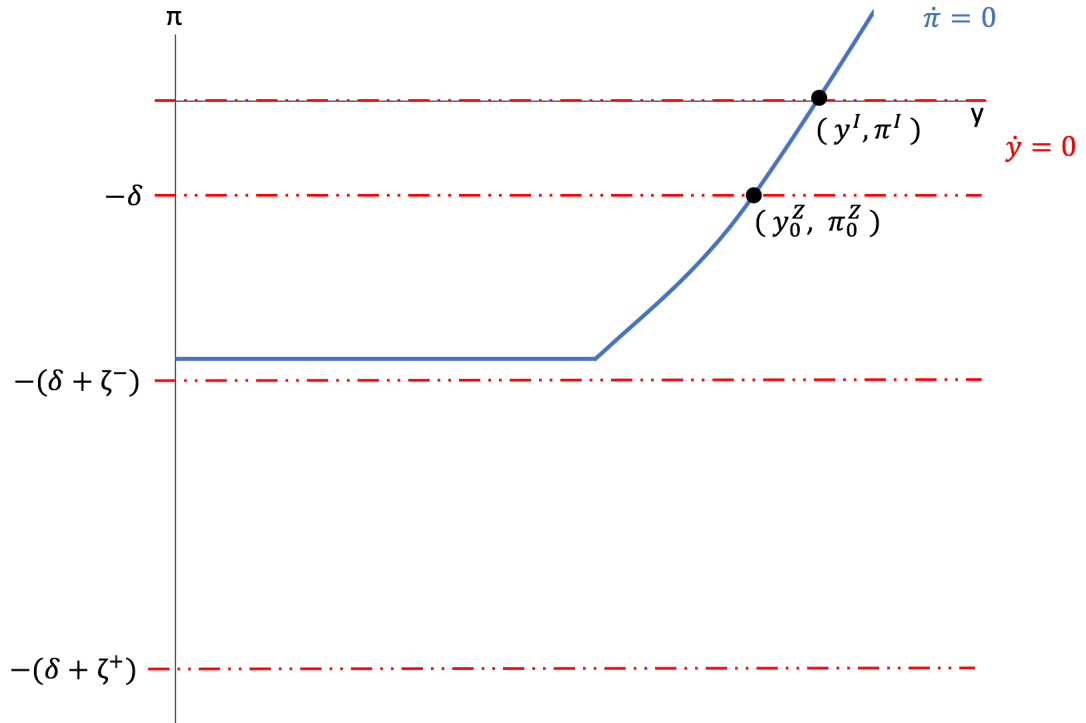


Figure 7: NIRP In Model With Wealth In Utility

