Aggregate Effects of Minimum Wage Regulation at the Zero Lower Bound *

Andrew Glover
University of Texas - Austin

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Abstract

The Fair Minimum Wage Act of 2007 increased the U.S. nominal minimum wage by 41 percent, just as interest rates hit the Zero Lower Bound. I study the interaction of these events in a parsimonious extension of the sticky-price New Keynesian model with heterogeneous labor. A "minimum-wage augmented" Phillips curve relates inflation to output and the real minimum wage, which I estimate with aggregate data. Consistent with theory, controlling for the real minimum wage reduces the effect of output on inflation and increasing the minimum wage is inflationary. I then calibrate the equilibrium model to match microeconomic elasticities of earnings with respect to the minimum wage. On aggregate, the ZLB's contractionary effects are dampened because nominal wages cannot decline rapidly, thereby halting the deflationary spiral caused by low aggregate demand. The effect can be large - in the calibrated economy, GDP losses from the ZLB are cut by half, even though only 3% of earnings accrue to minimum wage earners. Furthermore, increasing the minimum wage at the ZLB is expansionary, generating expected accumulated output gains of 14%.

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How do aggregate output and employment respond to increases in the minimum wage? How does the minimum wage interact with shocks to the economy? These are fundamentally macroeconomic questions, since the nominal minimum wage has general equilibrium effects on inflation and income, to which monetary policy responds. They are particularly relevant following the Great Recession, which triggered an unprecedented period of near-zero nominal interest rates, but was also preceded by a historically large increase in the federal minimum wage (41.8% nominally).[1] While the aggregate effects of nominal interest rates hitting the zero lower bound (ZLB) has since been extensively analyzed, the interaction between the ZLB and the nominal minimum wage has not. I study this interaction in a parsimonious extension of the sticky-price New Keynesian model that includes heterogeneous workers and nominal minimum wages.

I introduce two key features to the standard sticky-price New Keynesian model. First, workers have heterogeneous labor inputs that enter production with different efficiencies and with a general elasticity of substitution. Second, the minimum wage binds for low-productivity workers, which causes their labor supply to be constrained by demand. The resulting macroeconomic model is familiar, but novel. The main difference between this model and the textbook sticky-price model is in the "minimum-wage augmented" Phillips curve. The presence of a binding minimum wage reduces the slope with respect to output and variation in the real minimum wage shifts the Phillips curve over time.

I test this theory by estimating the Phillips curve with and without the minimum wage. I find strong support for the model in multiple dimensions. As predicted, the coefficient on the minimum wage is always positive and statistically significant. Furthermore, the coefficient on the output gap falls when the minimum wage is included and the model's fit improves. Finally, the coefficient on the minimum wage is larger when the model is estimated on data including the Great Recession, which is expected since the 2007 Fair Minimum Wage Act was a historically large increase in the minimum wage followed by a period of unexpectedly high inflation (given the output gap).

The minimum-wage augmented Phillips curve is supported by the data, but cannot be used for counterfactuals directly because the nominal minimum wage affects both expected and realized inflation (and therefore the path of the real minimum wage). I therefore use the full dynamic model for counterfactuals. I calibrate the model to match microeconomic earnings responses to minimum wage changes for workers throughout the wage distribution. The

[1] This was the largest increase since the 1970s. In real terms, it was much larger since inflation was much lower in the late 2000s.

counterfactual experiments show that monetary policy's interaction with the minimum wage is key to understanding its aggregate and distributional effects. When monetary policy dictates that nominal rates respond more than one-for-one to inflation, the model predicts that increasing the minimum wage is contractionary. However, if the central bank is willing to accept more inflation in response to a minimum wage increase then output rises with the minimum wage. Since monetary policy's response is so important, the minimum wage is particularly interesting when nominal interest rates are unresponsive to inflation, such as a ZLB episode.

I therefore consider a demand-driven recession, a la Eggertsson and Woodford [2004], in which the central bank lowers nominal interest rates to zero. The existence of a binding minimum wage dampens the negative output and inflation effects of such a shock by halting the deflationary spiral that typically causes a severe contraction during a ZLB episode. This is because *price* deflation during a recession requires even faster nominal wage deflation, which the minimum wage forbids (for some workers). When nominal wages remain high, firms cut prices less deeply, even in the face of low demand. For a sufficiently persistent ZLB shock, the gains from having any positive share of workers earn a binding minimum wage are unbounded.[2] Modeling the minimum wages therefore extends the model's range of application considerably, since the standard model becomes pathological before the ZLB shock is fully persistent.[3] Therefore, I also study the minimum wage in an extension of the standard model with wealth in utility, as in Michaillat and Saez [2018], which remains determinate even for a fully persistent ZLB shock under some parametric restrictions.

Quantitatively, accounting for the minimum wage brings the calibrated model closer to the 2009-2015 US experience. The minimum wage is an effective guard against the ZLB's deflationary spiral: in the calibrated economy, minimum-wage workers receive only 3% of earnings, yet the contractionary effect of the ZLB is reduced by nearly half and deflation is reduced by six percentage points on impact. This reduction in deflation is apropos. As documented by Ball and Mazumder [2011] and Hall [2011], the Great Recession did not bring rapid deflation, as predicted by both traditional Phillips Curve based forecasts and the standard New Keynesian model. While the model with a minimum wage still generates deflation, it brings the model closer to the US

[2] There is a discontinuity at zero - if the share of minimum wage earners is zero then the model simplifies to the standard representative agent economy.

[3] For example, in my baseline calibration the ZLB is expected to last for ten quarters. In the model without a minimum wage, the ZLB must be expected to bind for under eleven quarters in order to obtain a unique stable solution.

data. The minimum wage is therefore complementary to other mechanisms that reduce deflation at the ZLB, such as financial frictions (see Del Negro et al. [2015] and Christiano et al. [2015]). The higher minimum wage also provides an independent empirical basis for Fratto and Uhlig [2014]'s finding that the price markup shocks have driven post-2008 inflation, as firms respond to higher nominal wage costs by raising prices even while facing low demand. Perhaps most related to the minimum wage mechanism here is Coibion et al. [2012], who consider a model in which all wages are subject to downward nominal rigidity. In that case, which my model nests, there is no deflation.

Increasing the minimum wage during a zero lower bound episode achieves further output and inflation gains, since the central bank does not follow the contractionary policy response dictated by a Taylor Rule. This effect is similar to inflationary policy at the ZLB, which is expansionary in the standard model due to the "Paradox of Toil," as introduced by Eggertsson [2010a]. By increasing expected price inflation, these shocks reduce the real interest rate, which boosts aggregate demand. As I discuss in Section 4.2, the minimum wage has the same effect, but also strengthens this mechanism in economies with borrowing constraints, when increasing the minimum wage is more expansionary than purely inflationary shocks because it directly increases aggregate demand for constrained households.[4] A growing literature finds that borrowing-constraints undermine both conventional and unconventional monetary policy because many households do not respond to the real interest rate. Kaplan et al. [2016] show the effect of borrowing constraints for the operation of monetary policy in normal times and McKay et al. [2015] show that forward guidance is less effective when households are borrowing constrained.

This paper is also related to the literature on nominal wage stickiness in general, since the minimum wage is a particular type of downward nominal wage rigidity. Erceg et al. [2000] introduce both upward and downward nominal wage rigidity using monopolistically competitive wage setting by households who face Calvo adjustment costs across workers. Daly and Hobijn [2014] show how a generalized version of this model flattens the Phillips Curve, which also occurs in my model, although I emphasize that variation in the real minimum wage also shifts the Phillips Curve over time. My model is most closely related to Schmitt-Grohé and Uribe [2016], who study a representative agent sticky price model with downward nominal wage rigidity, in which household labor supply is constrained by labor demand in equilibrium.

[4]Including the minimum wage also expands the set of parameters that admit a determinate equilibrium at the zero lower bound. The minimum wage economy is determinate as long as the ZLB has finite expected duration, whereas the standard model is indeterminant when the ZLB episode is highly persistent.

However, the minimum wage is a unique form of nominal rigidity in two ways. First, it directly affects only a small share of the labor force, whereas existing models of downward nominal wage rigidities typically constrain all (or most) workers. I show that even if few workers are directly constrained by downward nominal wage rigidities, the interaction with deflationary shocks can generate large aggregate effects, especially when their real wages spill over to other workers through relative labor demand. Second, and more importantly, the minimum wage is a policy instrument that can be judiciously adjusted in response to aggregate shocks, whereas the existing literature treats wage rigidity as exogenous and policy invariant. To my knowledge, I present the first equilibrium model to study nominal wage rigidities that are directly controlled by policy.[5]

The paper proceeds as follows. I first derive a "four-equation" dynamic equilibrium model, including the minimum-wage augmented Phillips curve. I then estimate this Phillips curve using aggregate data on inflation (actual and expected), the output gap, and minimum wages. I calibrate the structural model to microeconomic data on earnings elasticities with respect to the minimum wage, which I then use to demonstrate the interaction between monetary policy and minimum wages, both in normal times and during a demand-driven recession when the zero lower bound restricts monetary policy. I discuss and finally conclude.

1 The Model

I study a parsimonious extension of the sticky-price New Keynesian model (Galí [2015]) with wealth in utility (WIU) ala Michaillat and Saez [2018] to allow for an infinite ZLB episode. I introduce two new features. First, labor is heterogeneous: workers differ in their labor efficiency and labor enters production with a general (constant) elasticity of substitution. Second, low-productivity workers are subject to labor supply constraints. These constraints will be determined by labor demand in equilibrium, which is a reduced form way to capture labor distortions of the minimum wage.

1.1 Households

A measure of large families comprise two types of workers, low and high productivity. Households choose individual workers' consumption and labor to

[5] Schmitt-Grohé and Uribe [2015] allow the downward wage constraint to respond to output according to an exogenous function, but not directly to policy.

maximize expected discounted utility and save in a risk-free nominal bond, the real value of which directly enters preferences with a constant marginal utility of $\Omega \geq 0$.[6] Expectations are taken with respect to two sources of uncertainty: increases in the nominal minimum wage and a preference shock which drives the interest rate on risk free bonds to the zero lower bound.

$$\max_{c,n_L,n_H} \mathbb{E} \sum_{t=0}^{\infty} \beta^t e^{\zeta_t} \left[\frac{\left(\int_0^1 c_{i,t}^{\frac{1}{\gamma}} di \right)^{\gamma(1-\sigma)}}{1-\sigma} - \Psi \frac{\nu}{1+\nu} \left(\epsilon_H n_H^{\frac{1+\nu}{\nu}} + \epsilon_L n_L^{\frac{1+\nu}{\nu}} \right) + \Omega \frac{b_t}{P_t} \right] \quad (1)$$

subject to:

$$\int_0^1 p_{i,t} c_{i,t} di + \frac{b_{t+1}}{1+i_t} = \epsilon_H W_{H,t} n_{H,t} + \epsilon_L W_{L,t} n_{L,t} + b_t + T_t, \quad (2)$$

$$n_{L,t} \leq \underline{N}_{L,t}. \quad (3)$$

This problem is standard except for the constraint $n_{L,t} \leq \underline{N}_{L,t}$. From the household's perspective, the upper bound on low-productivity labor is an exogenous random variable, but in equilibrium it will be set to labor demand (per worker). This assumption is directly comparable to the labor supply constraint in Schmitt-Grohé and Uribe [2016], except with heterogeneous labor and the assumption that only the low-productivity constraint binds. I show that the same equilibrium can be obtained from an extension of the search and matching framework of Blanchard and Gali Blanchard and Gali [2010] in Appendix A.

Household demand of each intermediate good $c_{i,t}$ is given by

$$c_i^d(p_{i,t}; P_t) = \left(\frac{p_{i,t}}{P_t} \right)^{\frac{\gamma}{1-\gamma}} C_t, \quad (4)$$

where P_t is the natural aggregate price index. The inter-temporal Euler Equation can then be written in terms of total consumption as

$$C_t^{-\sigma} = \beta \mathbb{E}_t \left[\frac{e^{\zeta_{t+1}}}{e^{\zeta_t}} (1+i_t) \frac{P_t}{P_{t+1}} \left(C_{t+1}^{-\sigma} + \Omega \right) \right]. \quad (5)$$

The expectation in this Euler Equation includes ζ_{t+1}, which will drive a demand shock and reduce the natural interest rate below zero. The household has two labor supply conditions, one for each worker type, and must respect

[6] This is without loss of generality, since nominal bonds are in zero net supply, so the marginal utility of wealth is constant, as in Michaillat and Saez [2018].

the upper bound on low-productivity labor. Per-worker labor supply is therefore

$$n^s_{H,t} = \left(\Psi^{-1}\frac{W_{H,t}}{P_t}C_t^{-\sigma}\right)^{\nu}, \tag{6}$$

$$n^s_{L,t} = \min\left\{\underline{N}_{L,t}, \left(\Psi^{-1}\frac{W_{L,t}}{P_t}C_t^{-\sigma}\right)^{\nu}\right\}. \tag{7}$$

1.2 Firms

A continuum of firms produce differentiated goods. These firms compete monopolistically by choosing prices, subject to Calvo [1983] price adjustment frictions, which makes the price decision forward looking. Firms compete monopolistically by choosing the price of their good, which they can reset with a constant probability in each period. The firm's price in a given period t determines the demand for their good, which they must meet by hiring labor. This leads the firm to choose labor inputs in the least costly way possible subject to their goods demand. The profit maximization problem for a price setting firm is:

$$\max_{p,N_L,N_H} \mathbb{E}_t \sum_{\ell=0}^{\infty} \lambda^{\ell}\frac{\beta^{\ell}e^{\zeta_{t+\ell}}C_{t+\ell}^{-\sigma}}{e^{\zeta_t}C_t^{-\sigma}}\left(pc^d_{i,t+\ell}(p;P_{t+\ell}) - W_{L,t}N_{L,t} - W_{H,t}N_{H,t}\right) \tag{8}$$

subject to:

$$c^d_{i,t+\ell}(p;P_{t+\ell}) \leq \left(\theta N_{L,t+\ell}^{\frac{\eta-1}{\eta}} + N_{H,t+\ell}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}, \forall \ell \geq 0 \tag{9}$$

Where $1-\lambda$ is the probability that a firm is allowed to change their price in a given period. The firm is owned by households, so the stochastic discount factor is derived from the household's marginal rate of substitution.

A firm that is not able to adjust its price in period t minimizes the cost of production to meet demand in that period, taking nominal wages as given. Given a price p in period $t+\ell$, the firm's labor demand conditions are given by

$$\frac{N^d_{i,H,t+\ell}}{N^d_{i,L,t+\ell}} = \left(\frac{W_{L,t+\ell}}{\theta W_{H,t+\ell}}\right)^{\eta}, \tag{10}$$

$$c^d_{i,t+\ell}(p;P_{t+\ell}) = N^d_{i,L,t+\ell}\left(\left(\frac{N_{H,t+\ell}}{N_{L,t+\ell}}\right)^{\frac{\eta-1}{\eta}} + \theta\right)^{\frac{\eta}{\eta-1}}. \tag{11}$$

Equation 10 implies that the ratio of labor inputs is independent of a firm's price, due to production exhibiting constant returns to scale, and the demand for high-productivity workers relative to low is positively related to the relative wage. If the relative wage of low-productivity workers rises by 1% (through minimum wage policy, for example) then the relative demand for low-productivity workers falls by η percent. Equation 11 then determines the scale of the firm's labor force based on total goods demand. The firm's price setting problem simplifies to:

$$\max_p \mathbb{E}_t \sum_{\ell=0}^{\infty} \lambda^\ell \frac{\beta^\ell e^{\zeta_{t+\ell}} C_{t+\ell}^{-\sigma}}{e^{\zeta_t} C_t^{-\sigma}} \Big((p - \mathcal{W}_{t+\ell}) c_{i,t+\ell}^d (p; P_{t+\ell}) \Big) \tag{12}$$

Where the nominal marginal cost of production, $\mathcal{W}_t$, is defined by combining Equations 10 and 11,

$$\mathcal{W}_t = \frac{W_{L,t}}{\theta} \left(\left(\frac{W_{L,t+\ell}}{\theta W_{H,t+\ell}} \right)^{\eta-1} + \theta \right)^{-\frac{1}{\eta-1}}. \tag{13}$$

1.2.1 Policy and Shocks

Monetary policy is governed by a Taylor Rule on nominal interest rates, which are subject to the zero lower bound. Policy responds when inflation differs from zero or output differs from steady state, i.e.

$$\log(1+i_t) = \max \left\{ -\log \left(\beta \mathbb{E}_t \frac{e^{\zeta_{t+1}}}{e^{\zeta_t}} \right) + \psi_\pi \log \frac{P_t}{P_{t-1}} + \psi_y \log \frac{Y_t}{Y}, 0 \right\}. \tag{14}$$

The minimum wage is modeled as the prevailing nominal wage for low-productivity workers, $W_{L,t}$ and follows the following stochastic process

$$\log W_{L,t} = \log W_{L,t-1} + m_t, \tag{15}$$

with the shock $m_t > 0$.

The shock to preferences follows a process with two regimes. In the first (non-ZLB) regime, $\zeta_{t+1} = \zeta_t$. In the second (ZLB) regime, the natural rate of interest is negative:

$$-\log \left(\beta \mathbb{E}_t \frac{e^{\zeta_{t+1}}}{e^{\zeta_t}} \right) = r^{zlb} < 0. \tag{16}$$

The probability of switching from the first regime to the second is zero while the probability of switching from the second to the first is $1 - \chi$.

1.3 Steady-State

In steady-state all real variables are constant, the inflation rate is zero, $\zeta = 0$ is constant and $m_t = 0, \forall t$. Since there is no inflation, the price-dispersion wedge is $\Delta = 1$. The list of steady-state equations can be found in Appendix B. The system is complicated by Equation 7. If the minimum wage does not bind in steady state then both households are on their labor supply curve and labor market clearing determines real wages (the price level is still indeterminate). If the low-productivity labor supply constraint binds then the low-productivity household is constrained and demand determines her labor in equilibrium, so policy must provide the final restriction for equilibrium. While the nominal minimum wage is the policy instrument away from steady state, the *real* minimum must be chosen to determine a steady-state equilibrium. I will linearize around the steady-state where the labor-supply constraint just binds, which means that

$$\frac{\epsilon_H}{\epsilon_L}\left(\frac{W_L}{\theta W_H}\right)^{\eta}\left(\Psi^{-1}\frac{W_H}{P}Y^{-\sigma}\right)^{\nu} = \left(\Psi^{-1}\frac{W_L}{P}Y^{-\sigma}\right)^{\nu}. \tag{17}$$

Since the nominal minimum wage process is increasing, I use the first term to linearize around the steady-state. The following analytical results are unchanged if the initial steady-state has a larger binding minimum wage.

1.4 Local Dynamics

Log-linearizing the economy around a steady-state with binding minimum wage yields a "four-equation" model

$$y_t = \xi \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}\big(i_t - r_t - \mathbb{E}_t \pi_{t+1}\big), \tag{18}$$

$$i_t = \max\{0, r_t + \psi_\pi \pi_t + \psi_y y_t\}, \tag{19}$$

$$w^r_{L,t} = w^r_{L,t-1} - \pi_t + m_t, \tag{20}$$

$$\pi_t = \frac{\nu(1-s_L)\kappa}{\eta s_L + \nu} y_t + \frac{\nu s_L \kappa(\eta+\nu)}{(1+\sigma\nu)(\eta s_L + \nu)} w^r_{L,t} + \beta \mathbb{E}_t \pi_{t+1}, \tag{21}$$

where r_t is the natural rate of interest at t, the WIU term $\xi \in (0,1]$ is decreasing in Ω, and $w^r_{L,t}$ is the real minimum wage.[7] The first equation is the New Keynesian IS curve (with $\mathbb{E}_t y_{t+1}$ discounted due to wealth in the utility function) and the second is a Taylor Rule representing monetary policy. The third equation is the law of motion for the real minimum wage, which follows from the processes for the nominal minimum wage and the price level. The final equation is the minimum-wage augmented Phillips curve, which is the most novel feature and warrants further discussion.

[7] See Appendix B for derivation of this system.

The Phillips curve depends on the composite price stickiness parameter κ, but also on the elasticity of substitution between labor types (η) and the fraction of output created by low-productivity workers (s_L). These two parameters dictate how this Phillips curve differs from the standard one without minimum wages. The elasticity parameter governs how much a change in $w_{L,t}$ passes through to $w_{H,t}$ in equilibrium. If the two worker types are perfect substitutes then $\eta = \infty$ and the weight on output in the Phillips curve goes to zero and the model is identical to a representative agent model in which the nominal wage is set by policy. When the two labor types are not perfect substitutes, the high-productivity wage does not respond one-for-one to the minimum wage, so the share of output created by each labor input matters. In that case, the larger is s_L the less output feeds back to inflation since fewer workers are on their labor supply curves. Finally, if the real minimum wage is set so that low-productivity workers' intratemporal Euler Equation holds then the Phillips curve collapses to the standard standard model without a minimum wage.

2 Empirical Evaluation

Before using the model to perform counterfactuals, I test the empirical relevance of minimum wages from the Phillips curve in Equation 21. My measure of inflation, π_t, is the annualized percentage growth in the CPI. I use the Michigan Survey of Consumers average inflation expectations for $\mathbb{E}_t\pi_{t+1}$ based on the findings of Coibion and Gorodnichenko [2015] and Coibion et al. [2017] that real-time household inflation expectations best proxy for $\mathbb{E}_t\pi_{t+1}$ in Phillips curve estimation. My measure of y_t is the percentage gap between real GDP and the CBO estimate of potential. In my baseline estimates, I control for real oil price growth as a supply shock.

My empirical measure of the real minimum wage is derived as follows: First, I create a time series of the average nominal minimum wage from state-level data provided by Neumark [2018], which I deflate by CPI. After taking logs of the real minimum wage series, I subtract the log of average labor productivity to create the minimum wage "gap".[8] Figure 1 plots the minimum wage gap used in my baseline regressions, as well as the unadjusted real minimum wage.

Table 1 reports estimates with $\pi_t - \mathbb{E}_t\pi_{t+1}$ as the dependent variable and the unemployment rate as the output measure. The first two columns use data from 1960Q1-2017Q3 while the third and fourth columns use data from 1960Q1

[8]Appendix E presents estimates from specifications using alternative measures of slack and the average real minimum wage as well as excluding oil prices. The results are similar.

until the onset of the Great Recession in 2007Q4. For each sample period, I begin by estimating the curve without the minimum wage (see Columns 1 and 3) and then add the minimum wage variable (Columns 2 and 4). In each case, the minimum wage affects the Phillips Curve in line with theory: it enters with a significantly positive coefficient, controlling for the minimum wage reduces the coefficient on the output gap, and the R^2 rises.[9] Furthermore, the coefficient on the minimum wage falls when I exclude the Great Recession period, which is expected since the minimum wage increased by a historically large amount just prior to the Great Recession, when inflation was high relative to the output gap. The estimates follow a similar pattern when I estimate the coefficient on $\mathbb{E}_t\pi_{t+1}$, as shown in Columns 5 and 6 of Table 1.

	Restricted $\beta = 1$				Unrestricted β	
Variable	(1) All Yrs	(2) All Yrs	(3) Excl. G.R.	(4) Excl. G.R.	(5) All Yrs	(6) All Yrs
M.W. Gap†		1.155***		0.716**		0.846***
		(0.304)		(0.300)		(0.241)
Output Gap	0.146**	0.073	0.060	0.029	0.135***	0.083
	(0.062)	(0.060)	(0.063)	(0.066)	(0.050)	(0.051)
Oil Price	0.054***	0.056***	0.040***	0.041***	0.044***	0.047***
	(0.015)	(0.016)	(0.011)	(0.011)	(0.016)	(0.017)
Exp. Inflation					1.313***	1.275***
					(0.054)	(0.048)
Obs.	232	232	191	191	232	191
R^2	0.246	0.315	0.148	0.182	0.817	0.827

Newey-West S.E. in parenthesis. †Coeff. and S.E. scaled by 100 for display precision.
*p<0.10; **p<0.05; ***p<0.01

Table 1: Phillips Curve Estimates

In summary, aggregate inflation responds to the minimum wage in a theoretically consistent and statistically significant manner. I conclude that the minimum-wage adjusted Phillips Curve is empirically relevant. I therefore use the full equilibrium model to measure the quantitative effect of minimum wages, especially when interest rates hit the zero lower bound due to low aggregate demand.

[9] The output gap coefficients have large standard errors, so I cannot say that their reduction is statistically significant.

3 Quantitative Evaluation

Calibrating this economy requires setting two new parameters, η and s_L, since together they determine the weights on output and the real minimum wage in the Phillips Curve.[10] Rather than use the coefficients from Table 1 for counterfactuals, I calibrate the elasticity of substitution using wage and earnings responses to changes in the minimum wage. I do this in order to give a conservative role for the minimum wage because my calibrated coefficient on the real minimum wage will be much smaller (relative to the coefficient on output) than implied by the estimates from Table 1. I find s_L from the share of earnings accruing to workers near the minimum wage.

For the elasticity of substitution, η, I use Equation 10 along with microeconomic estimates of the effect of increasing the minimum wage on earnings for workers earning wages at and above the minimum. This equation relates changes in minimum-wage earnings (relative to other workers) to changes in their relative wages:

$$\Delta \log \left(W_{L,t}N_{L,t}\right) - \Delta \log \left(W_{H,t}N_{H,t}\right) = (1-\eta)\Bigg(\Delta \log W_{L,t} - \Delta \log W_{H,t}\Bigg). \quad (22)$$

The moments required to compute η are reported in Neumark et al. [2004]. They estimate the response of wages and earnings to a 1% increase in the minimum wage for workers earning up to 800% of the minimum. Their estimates imply that workers up to 110% of the minimum wage are essentially perfect substitutes with minimum-wage earners, but workers become less substitutable with minimum-wage earners as their wages rise. Above 200% the elasticity fluctuates around $\eta = 0.80$, which is the value that I use in the following quantitative exercises (see Appendix F for the elasticity of substitution by earnings group).

I use data on earnings shares to calculate s_L, under the assumption that the minimum wage just binds for those earning it in steady-state.[11] In a steady-state where the minimum wage just binds, the natural moment for s_L is the share of earnings accruing to minimum wage workers. However, the effect of the minimum wage depends on the earnings share of workers for whom it binds *plus the earnings share of workers who are perfect substitutes for these workers.*

[10]For my baseline results, I study the model without wealth in utility, which means that $\xi = 1$. I will calibrate $\xi < 1$ when studying a permanent zero lower bound episode in Section 3.2.

[11]Note that this does not require that the minimum wage bind for these workers in steady state, just that any increase from steady state would cause it to bind.

Using Neumark et al. [2004]'s estimates of earnings shares for workers earning up to 110% of the minimum wage gives $s_L = 0.03$.

I set the parameters $\sigma = 2, \nu = 1, \beta = 0.99$ and $\kappa = 0.02$ so that my results are comparable to previous quarterly calibrations of the sticky-price model. I begin with active parameters for the Taylor Rule parameters, $\psi_\pi = 1.5$ and $\psi_y = 0.125$ and explore the effects of a less responsive monetary policy when $\psi_\pi = 0.75$. The interest-rate response to inflation, ψ_π, is pivotal for understanding the effects of a minimum wage increase.

3.1 Effects of Minimum Wage Away From ZLB

Figure 2 plots the impulse responses of aggregate output (consumption), inflation, and interest rates to a 40.8% increase in the nominal minimum wage for each level of ψ_π.[12] The solid line shows the aggregate effect of increasing the minimum wage when monetary policy responds strongly to inflation. Inflation increases as firms raise their prices in face of a higher nominal unit cost of production, which induces the central bank to raise the nominal interest rate. The net effect is an increase in the annual real rate, which reduces aggregate demand and causes output to fall. Over time both the wage of high productivity workers and the price level rise by 40.8% (so that relative labor inputs and the real unit cost of production both return to steady-state).

The contraction in output is entirely due to the Taylor Rule responding sharply to inflation, as shown by the dashed line in Figure 2 (for which $\psi_\pi = 0.75$). Increasing the minimum wage now causes more expected inflation so that the real rate falls on net, which increases aggregate demand and causes output to rise above steady state. The return to steady-state is much more rapid in this case, since the central bank accepts higher inflation.

These experiments show that the central bank has ultimate power in shaping the macroeconomy's response to increases in the minimum wage. I now study the effect of the minimum wage on output and inflation dynamics at the ZLB, when monetary policy is unresponsive to changes in the minimum wage.[13]

[12]This is a large shock relative to typical business cycle fluctuations, but to a small share of the population so the direct aggregate effect is much smaller. I will use first-order approximations throughout.

[13]While my focus is on the aggregate effects of increasing the minimum wage, there are also distributional effects on wages, hours, and earnings. As with aggregates, the monetary response and state of the economy are important, as I show in Appendix D.

3.2 Interaction of ZLB and Minimum Wage

I now study output and inflation when the economy starts in the ZLB regime. The economy is linearized around the non-ZLB regime steady-state, so that the log-linearized variables all return to zero when the ZLB episode ends.[14] I set the probability of the ZLB persisting to $\chi = 0.9$ so it is expected to last for ten quarters.

The system during the ZLB period is described by

$$y_t = \xi \chi y_{t+1} + \sigma^{-1} r^{zlb} + \sigma^{-1} \chi \pi_{t+1}, \tag{23}$$

$$\pi_t = \frac{\nu\kappa(1 - s_L)}{\eta s_L + \nu} y_t + \frac{\nu s_L \kappa(\eta + \nu)}{(1 + \sigma\nu)(\eta s_L + \nu)} w^r_{L,t} + \beta\chi\pi_{t+1}, \tag{24}$$

$$w^r_{L,t} = w^r_{L,t-1} - \pi_t. \tag{25}$$

For my baseline calibration, I use parameter values from Eggertsson and Woodford [2004] (henceforth, EW) and eliminate wealth in utility by setting $\xi = 1$. Writing this system in state-space form, the transition matrix has two eigenvalues outside of the unit circle and one inside. The equilibrium paths of output, inflation, and the real minimum wage are then determined by setting an initial condition on the real minimum wage, $w_{L,0} - p_0$, which is provided by the policy for the nominal minimum wage (since $p_0 = 0$).

The model exhibits a discontinuity at $s_L = 0$, which is worth discussing at this point, since one may wonder whether restricting $w^r_{L,\infty} \equiv \lim_{t\to\infty} w^r_{L,t}$ to be finite is economically necessary. If $s_L = 0$ then Equation 25 can be satisfied with $w^r_{L,\infty} = \infty$, but finite solutions of Equations 23 and 24 still exist. As soon as $s_L > 0$, $w^r_{L,\infty}$ must remain finite. This raises accuracy concerns about the local approximation around a steady state with $s_L > 0$ if raising the real minimum wage sufficiently high reduces low-productivity labor demand to zero over time. While this does not happen in the calibrated economy with $\eta < 1$ (since low-productivity labor's marginal product grows without bound as its input approaches zero), equilibria that converge to the homogenous-worker-flexible-price model are possible if $\eta > 1$. Since my calibrated $\eta < 1$, I study the equilibrium with a finite real minimum wage.

If the minimum wage just binds for low-productivity workers at the onset of the ZLB episode (as would be the case starting in steady state) and the nominal minimum wage is held constant, then the initial condition is sets the $t = 0$ real wage equal to the negative of inflation. The resulting path of output and inflation is labeled "Const. Min. Wage" in Figure 3, which shows that

[14] This includes the real minimum wage, which means that the nominal minimum wage is adjusted by the total change in the price level once the ZLB episode ends.

the economy still experiences a deflationary recession, the depth is greatly reduced relative to the economy without a minimum wage. Inflation falls to −4.8% on an annual basis and output losses are 7.4% on impact, which is large in absolute terms but roughly half of the decline without a minimum wage. Furthermore, inflation and output rise over time in this economy, which means that accumulated output losses over ten quarters amount to 72.5% with the minimum wage (rather than 143% in EW model).

Contrast this to the model of EW with fully flexible nominal wages, which (for this calibration) has a unique jump solution with rapid deflation of 10.5% per year and a deep recession (output falls by 14.3%). In that model, output and inflation jump to their ZLB values and are constant throughout the episode. The real wage falls by 43.03%, or three times output (since $\frac{1+\sigma\nu}{\nu} = 3$). This requires the nominal wage to fall by 45.7% at the onset of the ZLB and then 10.5% per year thereafter.[15] In expectation, accumulated output losses in the EW model are substantially higher than with the minimum wage (143.4% vs 72.5%) and realized gains from the minimum wage grow with ZLB duration, since the minimum wage causes inflation to converge to zero and flow output losses to converge to −2.5%.

As the ZLB episode persists in the minimum wage economy, $\pi_t \to 0$ according to Equation 25, and output converges to:

$$y^{mw} \equiv \lim_{t\to\infty} y_t = \frac{\sigma^{-1} r^n}{1-\chi} \tag{26}$$

This limit is independent of the share of minimum wage workers and finite for $\chi < 1$. This contrasts sharply with the EW model, where output jumps to

$$y^{ew} = \frac{\sigma^{-1} r^n}{1-\chi-\frac{\sigma^{-1}\kappa}{1-\beta\chi}}. \tag{27}$$

Furthermore, the output gap in the EW economy asymptotes to $-\infty$ as the persistence of the ZLB episode rises towards the value $\chi^* < 1$ defined by

$$\kappa = (1-\chi^*)(1-\beta\chi^*)\sigma(\chi^*)^{-1}. \tag{28}$$

The minimum wage therefore has a larger dampening effect as $\chi \to \chi^*$, since output losses are unbounded without it.

The pathological behavior of the baseline model at χ^* makes for a dramatic example, but it is useful to consider the minimum wage in the model

[15] The heterogeneous labor economy without a minimum wage has the same response as the homogeneous labor economy (the solution can also be obtained by fixing $w_{L,t} - p_t = -43.03\%$ in the above system).

with wealth in utility, which admits a finite and determinate equilibrium for a permanent ZLB episode, for sufficiently low values of ξ and β (see Appendix C). I set β and ξ such that the WIU model *without a minimum wage* replicates the output losses and deflation of the EW model, then plot the paths for inflation and output in the WIU with a binding minimum wage. The lines labeled "WIU Const. Min. Wage" in Figure 3 show that output losses and deflation are further dampened by the minimum wage in the WIU model, relative to the EW model with the same declines in output and inflation.

To understand the difference between these economies with and without the minimum wage, consider the firm's price setting problem during the ZLB period. In the standard model, low demand induces firms to lower prices, which is optimal because the nominal wage falls even more than prices. This raises the real interest rate, which further depresses demand. When the minimum wage restricts nominal wage deflation, this cycle is interrupted: firms do not cut prices as deeply because their nominal unit cost does not fall as much.[16] Therefore, prices deflate by less and real rates do not rise as severely, so the demand effects are reduced.

This exercise shows that the ZLB's negative consequences are diminished greatly even if a small share of the population is constrained by the minimum wage, but the U.S. actually raised the minimum wage just prior to interest rates hitting zero. I now use the model as a laboratory to ask what would have happened if the Fair Minimum Wage Act of 2007 had not been passed at that time.

3.3 Raising the Minimum Wage At The ZLB

Increasing the minimum wage during a ZLB episode is especially effective since the nominal interest rate does not rise in response. I show this by increasing the nominal minimum wage at the onset of the ZLB episode and keeping it constant until the ZLB episode ends (at which point I return the real minimum wage to steady-state).[17]

The effect of increasing the nominal minimum wage is plotted as the lines

[16] The nominal wage of high-productivity workers falls during the ZLB episode, which causes firms to shift demand towards high-productivity labor. Therefore, the unit nominal cost of production falls in this model, but by less than without a minimum wage.

[17] In Appendix G, I conduct an experiment in which the nominal minimum wage is indexed to the price level during the ZLB in order to increase the *real* minimum wage by $w_L^r\%$. While this policy was not considered on a federal level, many states have increased their minimum wages in recent years, so this is a simple way of capturing the expectation of further increases in minimum wages that may have affected price setting during the ZLB episode.

labeled “Higher Min. Wage” in Figure 3, which shows the differences in output and inflation between the economy with a 40.8% increase in the nominal minimum relative to the economy with $w_{L,0} = 0$. Increasing the minimum wage by 40.8% at the onset of the ZLB reduces output losses by 1.5%−pts at onset and cuts deflation by a similar amount. The relative output gains tend towards zero as the the ZLB persists, though the output gain remains around 1.4% after ten quarters (the expected duration of the ZLB episode) and the accumulated output gain over ten quarters is 14.4%. As above, the effects of increasing the minimum wage are larger in the WIU model with a permanent ZLB shock, as demonstrated by the lines labeled “WIU Higher Min. Wage” in Figure 3.

4 Discussion

Before concluding, I discuss how the minimum wage relates to previous research on the zero lower bound, as well as how it affects economies for which the complete-markets IS curve is violated.

4.1 The “Missing” Deflation

As discussed by Ball and Mazumder [2011] and Hall [2011], standard Phillips curves predicted much greater deflation than what was observed, given the large decline in output during the Great Recession. I have shown that the minimum-wage augmented Phillips Curve generates a smaller decline in both output and deflation than the standard model *for a given* shock to the real rate. I now show that deflation is reduced *for a given decline in output.* While Section 2’s estimates of the minimum-wage augmented Phillips Curve hinted at this result, the entire model allows for inflation expectations to adjust to the minimum wage and for the real minimum wage to follow the endogenous path implied by inflation, both of which require the entire model. I do this by solving for output and deflation dynamics in different models: 1) the EW model without a minimum wage, 2) the model with a constant minimum wage, and 3) the model with a 40.8% increase in the minimum wage. In each case, the initial decline in output is exactly 10%, which is in line with output losses (from trend) during the Great Recession (see Glover et al. [2011]).

The inflation responses are plotted in Figure 4 for the EW model as well as two calibrations of the minimum wage model. The EW model (“EW, No Min. Wage”) has a constant deflation rate of just under 7.5% on an annual basis, while the model with a constant minimum wage and parameter values from the baseline calibration ("Const. Min Wage") reduces the initial deflation by 1% and 1.5% after ten quarters. Increasing the minimum wage by 40.8% at the onset of the ZLB episode ("Higher Min. Wage") reduces deflation by a further 0.5% relative to the constant case.

The highest two lines correspond to a calibration with s_L and η set so that the Phillips Curve parameter on the real minimum wage is 10% as large as the term on output, which is in line with my estimates from Section 2.[18] Deflation is further reduced: by roughly 2% relative to the baseline economy when the minimum wage is held constant ("Const. Min. Wage, PC Estimates") and by 4% when it is raised at the onset of the ZLB episode ("Higher Min. Wage, PC Estimate").

4.2 Paradox of Toil

Eggertsson [2010a] introduced the "Paradox of Toil", which demonstrates that inflationary shocks to the labor market can be expansionary at the zero lower bound, even if those shocks are contractionary in normal times (Eggertsson [2010a] considers labor supply). In this model, increasing the minimum wage at the ZLB is expansionary because it increases inflation, which reduces the real interest rate. However, the effect is complicated because the real minimum wage responds endogenously to the inflation caused by increasing the minimum wage. The aggregate effect of this feedback was not previously known and has been debated in discussions of the Paradox of Toil. Mulligan [2011] argued that increasing the minimum wage as a test of the Paradox of Toil, while Eggertsson [2010b] claimed that it was not a valid test because the counterfactual is unobservable and because minimum wages have differential effects across the wage distribution. My model has been constructed so that the minimum wage directly affects only low-wage earners. I have shown that the model's prediction for inflation (the minimum-wage augmented Phillips Curve) is supported empirically and used the remaining structure to construct the counterfactual. I conclude increasing the minimum wage is expansionary at the zero lower bound.

4.3 Effect of Borrowing Constraints

Demand is determined by the intertemporal Euler Equation in the model presented thus far, which means that the minimum wage's effect on output is

[18] The baseline calibration has a much smaller weight on the real minimum wage relative to the weight on output - about 2% as opposed to 10%.

purely through real interest rates. However, a growing literature argues that much of the real effect of monetary policy arises through labor markets and income effects, because many households are borrowing constrained or otherwise behave counter to the intertemporal Euler Equation (see Kaplan et al. [2016] and McKay et al. [2015]). I therefore enrich the model to include this mechanism by introducing a share of households who have high productivity, but consume in a hand-to-mouth fashion. For these households, consumption solves the intra-temporal Euler Equation

$$\left(C_t^{HM}\right)^{-\sigma}\frac{W_{H,t}}{P_t} = \Psi\left(C_t^{HM}\right)^{\frac{1}{\nu}}\left(\frac{W_{H,t}}{P_t}\right)^{-\frac{1}{\nu}}, \tag{29}$$

and thus, using the equilibrium expression for $\frac{W_{H,t}}{P_t}$, the hand-to-mouth household's consumption deviates from steady state according to

$$c_t^{HM} = \left(\frac{1+\sigma\nu}{\eta s_L+\nu}\right)y_t + \left(\frac{1+\nu}{1+\sigma\nu}\right)\left(\frac{\eta s_L}{\eta s_L+\nu}\right)w_{L,t}^r. \tag{30}$$

The remaining household's consumption still follows from their intertemporal Euler Equation, and total consumption is just the sum of the two. This means that increasing the minimum wage during the ZLB still affects output through demand, but now in two ways. As before, it creates inflation, which decreases the real interest rate, spurring demand by households who are on their Euler Equations. But now, it increases wages directly, which raises consumption demand for hand-to-mouth households.

The presence of hand-to-mouth consumers augments the IS curve in this economy. Denoting the steady-state share of consumption of hand-to-mouth households by s_c, the log-linearized inter-temporal Euler Equation for all other households can be written in terms of output and the real minimum wage as

$$y_t = \mathbb{E}_t y_{t+1} - s_c\Gamma_w\left(\mathbb{E}_t w_{L,t+1}^r - w_{L,t}^r\right) - (1-s_c)\Gamma_R\left(i_t - r_t - \mathbb{E}_t\pi_{t+1}\right), \tag{31}$$

where the constants are given by

$$\Gamma_w = s_c\left(\frac{1+\nu}{1+\sigma\nu}\right)\left(\frac{\eta s_L}{\eta s_L+\nu}\right)\left(1 - s_c\frac{1+\nu}{\eta s_L+\nu}\right)^{-1}, \tag{32}$$

$$\Gamma_R = (1-s_c)\sigma^{-1}\left(1 - s_c\frac{1+\nu}{\eta s_L+\nu}\right)^{-1}. \tag{33}$$

This IS curve is similar to the "discounted Euler Equation" introduced by McKay et al. [2016], which itself is a reduced form for borrowing constraints as studied by McKay et al. [2015]. When some households are constrained, the ZLB episode is directly dampened because the preference shock only reduces demand for a fraction of the population. It also dampens the feedback from deflation to output, since the consumption of hand-to-mouth households does not respond to the real interest rate. These two mechanisms reduce the severity of the ZLB episode in the McKay et al. [2015] model, while also reducing the effect of inflationary shocks to the Phillips Curve since, again, output responds less to changes in the real interest rate. The novelty of my model is that the real wage of high-productivity workers determines demand for hand-to-mouth households. On the one hand, this exacerbates the ZLB contraction because wages (and therefore earnings) fall rapidly with output (the $\frac{1+\sigma\nu}{\eta s_L+\nu}y_t$ term in Equation 30). On the other hand, the real minimum wage rises during the ZLB episode, which increases demand for these households (the second term in Equation 30).

In order to assess the effect of hand-to-mouth households' demand, I fix the share of consumption for hand-to-mouth households at $s_c = 0.133$, which corresponds to the lower bound for the share of wealth held by hand-to-mouth households reported by Kaplan et al. [2014]. I then solve the hand-to-mouth model both with a constant nominal minimum wage and an increase of 40.8%, as in the baseline model. As plotted with the line labeled "Hand-to-Mouth" in Figure 5, deflation and output losses are larger with hand-to-mouth households, which is due to a larger fall in demand for hand-to-mouth households because their wages initially fall. However, as the ZLB episode persists, the real minimum wage rise increases, which increases the earnings of hand-to-mouth households and, therefore, their consumption demand. This causes the paths of output and inflation to be steeper in the model with hand-to-mouth households than in the baseline (labeled "Baseline" in Figure 5).

While hand-to-mouth households cause larger output losses during the ZLB, they increase the expansionary effect of raising the nominal minimum wage during the ZLB. This can be seen from the lines labeled "Higher MW, H2M" in Figure 5 relative to the "Higher MW, Baseline" lines. In the model without hand-to-mouth households, increasing the minimum wage reduces accumulated output losses by 14.4% over ten quarters (the expected duration of the ZLB), which increases to 22.9% in the model with hand-to-mouth households.

The presence of hand-to-mouth households also interacts with the Paradox of Toil, because raising the minimum wage has a larger effect in the hand-to-mouth model than an equivalent inflationary shock to the Phillips Curve.[19]

[19]Such an inflationary shock is modeled as a constant $\tau > 0$ added to the right-hand side

The lines labeled "Inflationary Shock, H2M" in Figure 5 show output and inflation during the ZLB episode with hand-to-mouth households when the Phillips curve is shocked by $\tau = 0.016$ in each period (this value is chosen so that the inflationary shock and minimum wage increase generate identical paths for inflation and output in the model without hand-to-mouth households). In the baseline model, an inflationary Phillips curve shock accumulates output gains through ten quarters by 14.4%, which is the same as when the minimum wage is increased. However, with hand-to-mouth households, these gains are less than the equivalent minimum wage increase ($\tau = 0.016$ accumulates gains of 22.0% rather than 22.9% for the minimum wage increase).

In summary, the minimum wage is more expansionary if borrowing constraints generate a direct demand effect of higher real wages in the IS curve. It is also more expansionary than previously considered inflationary shocks to the Phillips Curve, suggesting that it may be a useful, if extremely unconventional, policy instrument during future ZLB episodes.

5 Conclusion

The minimum wage is a perennial topic of policy debate and the sticky-price New Keynesian model provides a natural framework to study its aggregate effects. I have presented a parsimonious extension of the textbook model which demonstrates that the macroeconomic effects of increasing the minimum wage are dictated by the state of the economy and monetary policy's response to inflation. In normal times, if monetary policy responds strongly to inflation by raising nominal interest rates more than one-for-one, then increasing the minimum wage is contractionary. However, when interest rates are stuck at zero due to a deep demand-driven recession, increasing the minimum wage raises inflation, but does not instigate a countervailing monetary response; it is therefore expansionary.

Even if the minimum wage is constant, introducing it into the model reshapes the economy's response to macroeconomic shocks. The minimum wage greatly reduces the severity of a demand-driven ZLB episode by dampening the deflationary effects of such demand shocks, thereby halting the spiral associated with the ZLB. While the share of minimum wage workers determines the extent to which the ZLB's ill effects are alleviated, any positive share of these workers has a large effect in the face of a sufficiently persistent ZLB shock. An economy with a realistically small share of minimum-wage earners reduces the negative effects of the ZLB by more than half.

of the Phillips Curve for the duration of the ZLB episode.

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Figure 1: Minimum Wage Measures, Log Difference From Series Averages

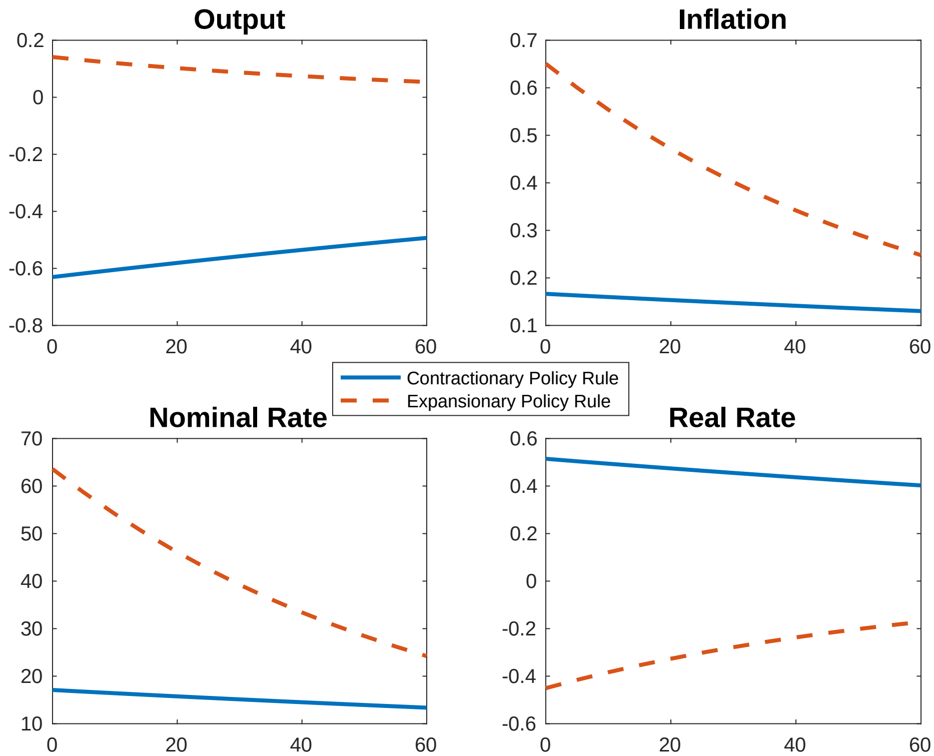
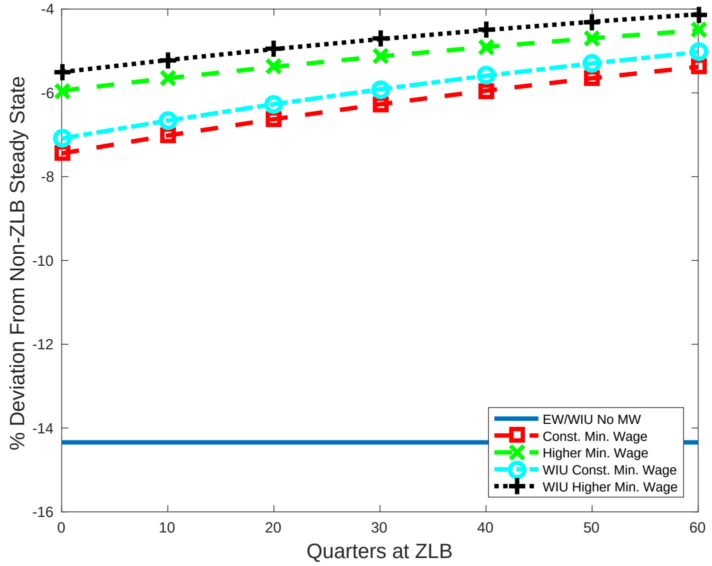
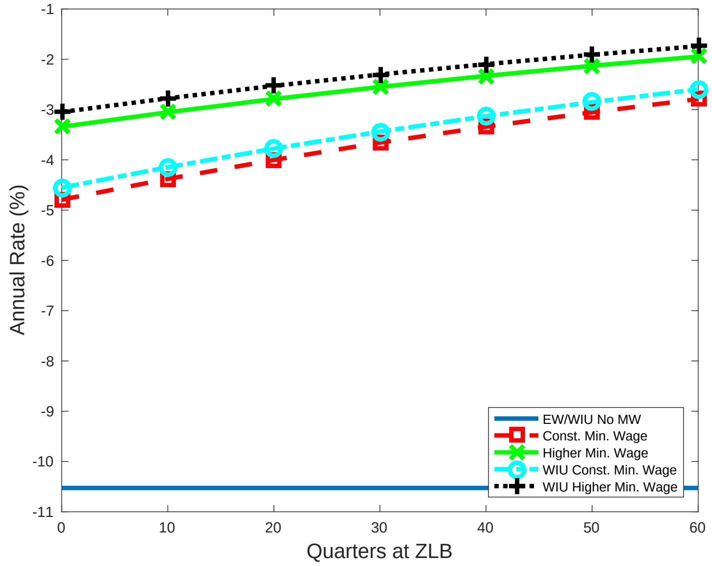


Figure 2: Aggregate Responses By Monetary Policy Response



(a) Output



(b) Inflation

Figure 3: Output and Inflation During ZLB Episode

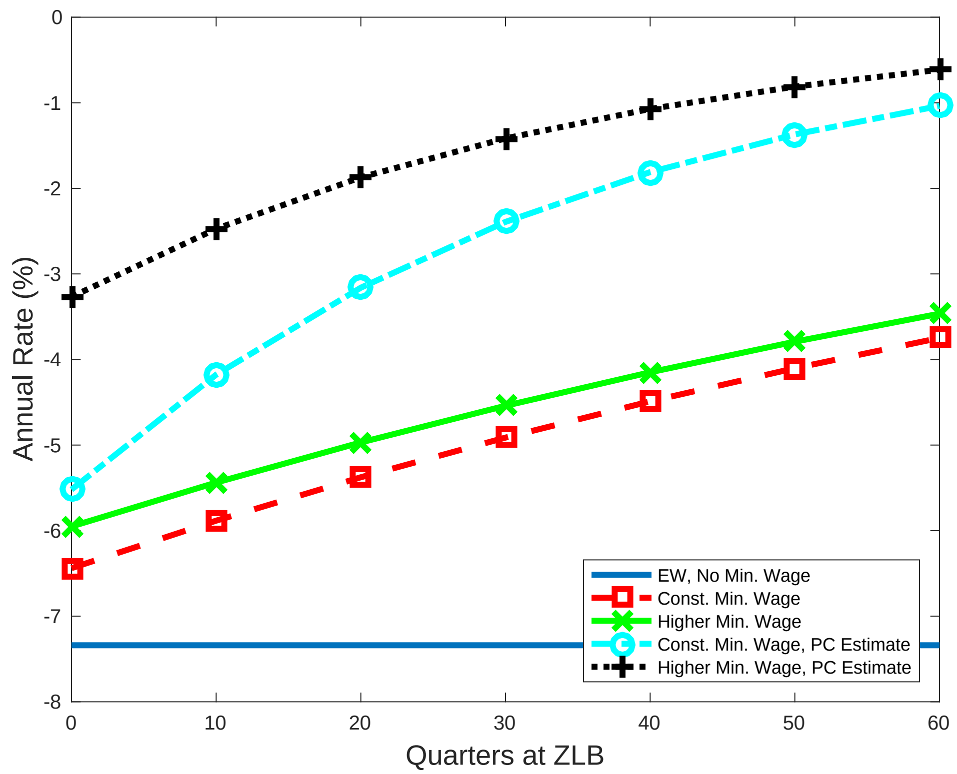
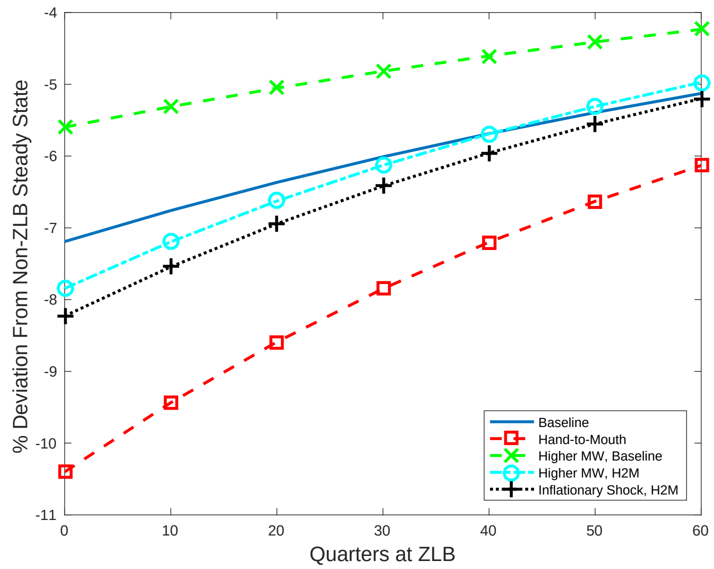
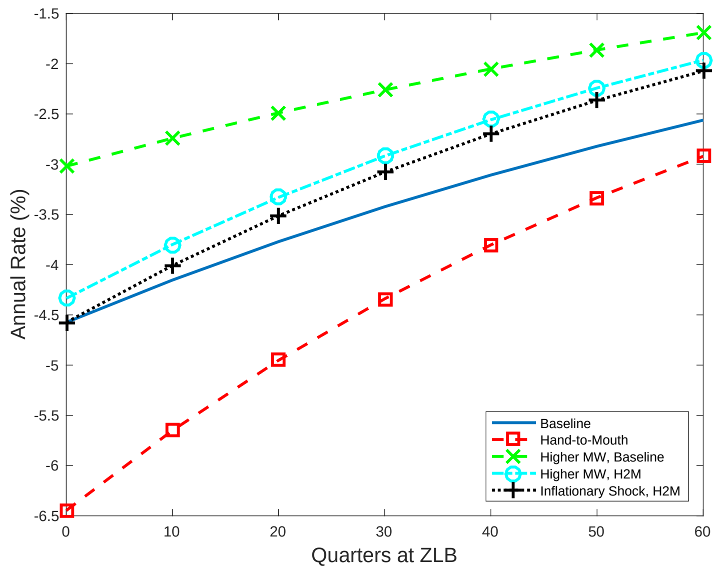


Figure 4: Predicted Inflation: Initial Output Decline of Ten Percent



(a) Output



(b) Inflation

Figure 5: Output and Inflation During ZLB Episode

AGGREGATE EFFECTS OF MINIMUM WAGE REGULATION AT THE ZERO LOWER BOUND
ONLINE APPENDIX
NOT FOR PUBLICATION

A Search and Matching

Suppose that firms must hire workers subject to matching frictions in each period and that a match, once made, lasts for exactly one period. Following Blanchard and Gali [2010], I assume that firms choose how many workers they wish to hire of each type, but incur a cost for each hire that depends on the aggregate ratio of hires to workers. For a firm i with labor demand for each worker type given by N_L and N_H, when total hiring for each worker type is $N_{L,t}$ and $N_{H,t}$, the total cost of hiring is given by

$$G_{i,t}(N_L, N_H) = B\left[\left(\frac{N_{L,t}}{\epsilon_L}\right)^{\alpha} N_L + \left(\frac{N_{H,t}}{\epsilon_H}\right)^{\alpha} N_H\right]. \tag{34}$$

A firm with price p solves the following cost minimization problem

$$\min_{N_L, N_H} w_{L,t}N_L + w_{H,t}N_H - G_{i,t}(N_L, N_H) \tag{35}$$

$$\text{s.t.} \tag{36}$$

$$c^d_{i,t}(p; P_t) \leq \left(\theta N_L^{\frac{\eta-1}{\eta}} + N_H^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}, \tag{37}$$

where the wages, $w_{i,t}$, differ from the competitive market from my baseline model as described below and are expressed in real terms. The first-order conditions for this problem give conditions similar to Equations 10 and 11. That is, the labor demand for firm i satisfies

$$\frac{N^d_{i,H,t}}{N^d_{i,L,t}} = \left[\frac{w_{L,t+\ell} + B\left(\frac{N_{L,t}}{\epsilon_L}\right)^{\alpha}}{\theta\left(w_{H,t+\ell} + B\left(\frac{N_{H,t}}{\epsilon_H}\right)^{\alpha}\right)}\right]^{\eta}, \tag{38}$$

$$c^d_{i,t}(p; P_t) = N^d_{i,L,t}\left(\left(\frac{N_{H,t}}{N_{L,t}}\right)^{\frac{\eta-1}{\eta}} + \theta\right)^{\frac{\eta}{\eta-1}}, \tag{39}$$

where the fact that all firms set the ratio of labor inputs to a constant value has been used in Equation 39.

Households have the same preferences as in my baseline model. The household takes the wage of each worker type as given, as well as the total number of hires in equilibrium. They can then choose how many workers of a given type look for work, but not exactly how many are employed. Denoting the total number of job-seeking workers of type j as $N^s_{j,t}$, the fraction of a given household's members of type j that are matched is given by $\frac{N_{j,t}}{N^s_{j,t}}$, which I ensure is less than (or equal) to one in steady state (and will assume to be less than one outside of steady state).

Real wages, $w_{j,t}$, are now determined through Nash Bargaining subject to the lower bound of $\frac{W_{L,t}}{P_t}$. When the minimum wage does not bind, a worker's real wage is given by Equation 15 in Blanchard and Gali [2010], otherwise it is equal to the real minimum wage. I further assume that worker's have zero bargaining power, which means that real wages are given by

$$w_{j,t} = \max\left\{\frac{W_{L,t}}{P_t}, \frac{\Psi N_{j,t}^{\frac{1}{\nu}}}{C_t^{-\sigma}}\right\}. \tag{40}$$

Under the assumption that high-productivity wages are above the minimum wage, this means that $N^s_{H,t} = N_{L,t}$, so all high-productivity workers seeking employment are matched with a firm. When the minimum wage, $\frac{W_{L,t}}{P_t}$, binds for low-productivity workers we have $N^s_{L,t} \geq N_{L,t}$, and therefore some of these workers are unemployed.

Finally, taking the limit of the above economy as $B \to 0$ yields the same equilibrium conditions as in my baseline model.

B Aggregate Equations

The system describing steady-state is as follows:

$$1 = \beta(1+i)\Big[1 + Y^{\sigma}\Omega\Big] \tag{41}$$

$$N_H = \left(\Psi^{-1}\frac{W_H}{P}Y^{-\sigma}\right)^{\nu} \tag{42}$$

$$N_L = \min\left\{\frac{\epsilon_H}{\epsilon_L}N_H\left(\frac{W_L}{\theta W_H}\right)^{\eta}, \left(\Psi^{-1}\frac{W_L}{P}Y^{-\sigma}\right)^{\nu}\right\} \tag{43}$$

$$\frac{\epsilon_L N_L}{\epsilon_H N_H} = \left(\frac{W_L}{\theta W_H}\right)^{\eta} \tag{44}$$

$$Y = \left((\epsilon_H N_H)^{\frac{\eta-1}{\eta}} + \theta(\epsilon_L N_L)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{45}$$

$$\frac{\mathcal{W}}{P} = \gamma^{-1} \tag{46}$$

$$\frac{\mathcal{W}}{P} = \left(\left(\frac{\theta W_H}{W_L}\right)^{\eta-1} + \theta\right)^{\frac{\eta}{\eta-1}}\frac{W_L}{P} \tag{47}$$

The steady-state with a just-binding real minimum wage is the starting point for analysis. I now derive the four equation model and the expressions for real wages, hours and earnings. To begin, I list the linearized equilibrium conditions:

$$y_t = \left(\frac{Y^{-\sigma}}{Y^{-\sigma}+\Omega}\right)\mathbb{E}_t y_{t+1} - \sigma^{-1}\big(i_t - r_t - \mathbb{E}_t\pi_{t+1}\big) \tag{48}$$

$$i_t = \max\{0, r_t + \psi_y y_t + \psi_\pi \pi_t\} \tag{49}$$

$$n_{H,t} = \nu(w_{H,t} - p_t) - \sigma\nu y_t \tag{50}$$

$$\phi_t = \eta(w_{H,t} - w_{L,t}) \tag{51}$$

$$n_{L,t} = \phi_t + n_{H,t} \tag{52}$$

$$n_{H,t} + s_L\phi_t = \delta_t + y_t \tag{53}$$

$$\delta_t = 0 \tag{54}$$

$$\omega_t = w_{L,t} - \mu\phi_t \tag{55}$$

$$p_t^{\#} = (1-\beta\lambda)\omega_t + \beta\lambda\mathbb{E}_t p_{t+1}^{\#} \tag{56}$$

$$p_t = \lambda p_{t-1} + (1-\lambda)p_t^{\#} \tag{57}$$

$$\pi_t = p_t - p_{t-1} \tag{58}$$

Where all variables are in percentage deviations from steady state except for those already expressed as rates, which are in absolute deviations. The term multiplying $\mathbb{E}_t y_{t+1}$ in Equation 48 arises from wealth in utility, and is labeled ξ in the draft. The new parameter μ is the elasticity of the marginal product of low-productivity workers with respect to their labor input evaluated in steady state. With the constant elasticity production function this parameter is constant and can be written as $\mu = \eta^{-1}(s_L - 1)$. The intermediate variable ϕ_t is the ratio of low to high productivity workers. Combining Equations 50, 51, and 53 gives the high-productivity real wage as a function of output and the real minimum wage:

$$w_{H,t} - p_t = \frac{1+\sigma\nu}{\eta s_L + \nu} y_t + \frac{\eta s_L}{\eta s_L + \nu}(w_{L,t} - p_t) \tag{59}$$

This then gives ϕ_t as:

$$\phi_t = \frac{\eta(1+\sigma\nu)}{\eta s_L + \nu} y_t - \frac{\eta\nu}{\eta s_L + \nu}(w_{L,t} - p_t) \tag{60}$$

Which then gives the real unit cost of production as:

$$\omega_t - p_t = \left(1 + \mu\frac{\eta\nu}{\eta s_L + \nu}\right)(w_{L,t} - p_t) - \mu\frac{\eta(1+\sigma\nu)}{\eta s_L + \nu} y_t \tag{61}$$

And finally, setting $\mu = \eta^{-1}(s_L - 1)$ and using Equations 56, 57, and 58 gives the Phillips Curve as in Equation 21. The labor supply and real earnings of each worker type are then recovered from the appropriate equations above.

C Wealth In Utility

I have included wealth directly in household utility, ala Michaillat and Saez [2018], but have a discrete time model as opposed to their continuous time setting. It is therefore useful to consider the model without minimum wages, but with $\xi < 1$. The Phillips Curve without a minimum wage is given by

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t \pi_{t+1}. \tag{62}$$

Away from the ZLB, The IS curve from Equation 48 can be written as

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma\xi}\left[\psi_\pi \pi_t + \Big((1-\xi)\sigma + \psi_y\Big) y_t - \mathbb{E}_t \pi_{t+1}\right], \tag{63}$$

which is the IS curve in a perturbed model without wealth in utility, but risk aversion of $\sigma\xi$ and a Taylor Rule with coefficient $\psi_y + \sigma(1-\xi)$ on output.

The condition for a unique stable equilibrium of the above model is a relaxed version of the Taylor Principle

$$\kappa(\psi_\pi - 1) + (1-\beta)\big[\psi_y + \sigma(1-\xi)\big] > 0, \tag{64}$$

which means that the model has a unique equilibrium even when $\psi_\pi < 1$, as long as β and ξ are sufficiently small.

This logic extends to the ZLB episode, in which case the standard model requires $\chi < 1$ for determinacy, but the wealth-in-utility model is determinate even for a permanent ZLB when ξ and β are sufficiently small. To see this, let $\chi = 1$ and $s_L = 0$, which means that the ZLB system becomes:

$$\pi_t = \kappa y_t + \beta \pi_{t+1} \tag{65}$$

$$y_t = \xi y_{t+1} - \frac{1}{\sigma}\Big(- \pi_{t+1} - r^{zlb}\Big). \tag{66}$$

The above system has two eigenvalues outside of the unit circle whenever $\sigma(1-\beta)(1-\xi) > \kappa$, in which case both inflation and output jump to

$$y^{WIU} = \frac{\sigma^{-1} r^{zlb}}{1 - \xi - \frac{\sigma^{-1}\kappa}{1-\beta}}, \tag{67}$$

$$\pi^{WIU} = \frac{\kappa y^{WIU}}{1-\beta}. \tag{68}$$

Clearly both y^{WIU} and π^{WIU} are negative whenever $\sigma(1-\beta)(1-\xi) > \kappa$ (i.e. whenever the model is determinate).

For my WIU simulations of a permanent ZLB episode, I set β and ξ so that the decline in output and inflation are the same as in the Eggertson-Woodford economy when $\chi = 0.90$. This requires setting:

$$\beta = 1 - \kappa \frac{y^{ew}}{\pi^{ew}}, \tag{69}$$

$$\xi = 1 - \frac{\sigma^{-1} r^{zlb}}{y^{ew}} - \frac{\sigma^{-1}\pi^{ew}}{y^{ew}}. \tag{70}$$

D Distributional Effects

While my focus is on the aggregate effects of increasing the minimum wage, there are also distributional effects on wages, hours, and earnings. As with aggregates, the monetary response and state of the economy are important.

The expressions for real wages, equilibrium labor, and earnings for each worker type are given by:

$$w_{H,t} - p_t = \frac{1+\sigma\nu}{\eta s_L + \nu} y_t + \frac{\eta s_L}{\eta s_L + \nu}(w_{L,t} - p_t) \tag{71}$$

$$n_{H,t} = \nu \frac{1-\sigma\eta s_L}{\eta s_L + \nu} y_t - \frac{\nu\eta s_L}{\eta s_L + \nu}(w_{L,t} - p_t) \tag{72}$$

$$e_{H,t} = n_{H,t} + w_{H,t} - p_t \tag{73}$$

$$n_{L,t} = n_{H,t} + s_L^{-1}(y_t - n_{H,t}) \tag{74}$$

$$e_{L,t} = n_{L,t} + w_{L,t} - p_t \tag{75}$$

Away from the zero-lower bound, increasing the minimum wage may have negative or positive effects on high-productivity workers since monetary policy will dictate the output response. Furthermore, output effects are important for $n_{H,t}$ and $w_{H,t}$ due to s_L being small. Figure 6 shows the effect of a 40.8% increase in the nominal minimum wage when the central bank responds strongly to inflation ($\psi = 1.5$). The real minimum wage rises initially and then returns toward steady-state as prices inflate. This is met with a large fall in low-productivity labor demand, both because the minimum wage makes low-productivity workers relatively expensive and because monetary policy creates a contraction.

High-productivity workers experience a temporary fall in real wages, due entirely to the contraction created by monetary policy: they experience a net decline in labor demand, even though their relative demand rises due to the minimum wage hike. This causes a small decline in high-productivity real wages and earnings. Taken together, the minimum wage increase reduces wage inequality by nearly the entire 40.8% upon impact, but the decline in earnings inequality is dampened by the shift in hours - the earnings gap falls by just over 8% on impact.

When the central bank responds less to inflation, the minimum wage increase is expansionary and the effects on high-productivity workers are reversed. Now there is an increase in absolute labor demand for these workers, which increases their wage and earnings in equilibrium. Low-productivity workers still experience a decline in labor demand, but this is much smaller than in the contractionary case. The net effect is a smaller reduction in the wage and earnings gap than in the contractionary case. The responses of real wages, hours, and earnings depend on the real minimum wage and output at a point in time as follows:

E Alternative Phillips Curve Estimates

I now estimate the Phillips Curve under different specifications. The first two columns of Table 2 use the unemployment gap instead of the output gap. As in my baseline estimates, the minimum wage variable is significant and the magnitude of the coefficient on the unemployment gap falls when the minimum wage is included. The third and fourth columns reproduce the baseline estimates, but without controlling for oil price growth: the same pattern as in the baseline is found. Finally, columns five and six use the log of the average real minimum wage instead of the real minimum wage relative to average labor productivity, which generates an even larger point estimate on the minimum wage relative to the output gap

F Elasticity of Substitution

Figure 8 shows each worker group's elasticity of substitution with minimum wage workers, according to the estimates from Neumark et al. [2004]. Since the elasticity fluctuates around 0.8 once workers make twice the minimum wage, I use this value in all quantitative exercises.

G Constant Real Minimum Wage

If the nominal minimum wage is indexed to the price level then $w^r_{L,t}$ is replaced with the constant w^r_L in Equation 24 and Equation 25 is dropped from the system. There are two values of w^r of particular interest. If $w^r = -43.03\%$ then the the ZLB causes the economy to jump to the EW solutions. If $w^r = 172.2\%$, then output losses from the ZLB are completely offset and there is actually inflation during the ZLB epsode (about 2% annually). As w^r varies between these two values there is a monotonic reduction in output losses and deflation, as seen in Figure 9.

Variable	(1) U. Gap	(2) U. Gap	(3) No Oil	(4) No Oil	(5) Real M.W.	(6) Real M.W.
Min. Wage Gap†		1.086***		1.048***		
		(0.305)		(0.352)		
Log Min. Wage†						2.361**
						(0.987)
Unemp. Gap	-1.673***	-0.913*				
	(0.535)	(0.530)				
Output Gap			0.178**	0.112*	0.146**	0.106
			(0.069)	(0.068)	(0.062)	(0.066)
% Ch. Oil Price	0.054***	0.056***			0.054***	0.055***
	(0.015)	(0.016)			(0.015)	(0.015)
Observations	232	232	232	232	232	232
R^2	0.262	0.320	0.0530	0.111	0.246	0.275

Newey-West S.E. in parenthesis. †Coeff. and S.E. scaled by 100 for display precision.
*p<0.10; **p<0.05; ***p<0.01

Table 2: Alternative Specifications of Phillips Curve

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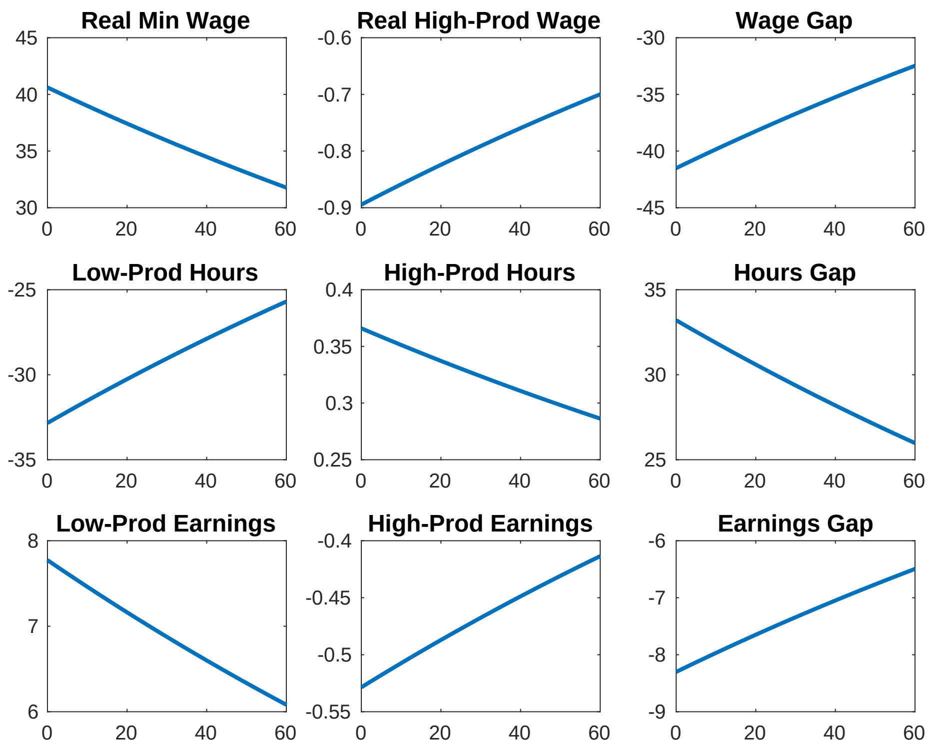

Figure 6: Distributional Effects: Contractionary Min Wage Hike

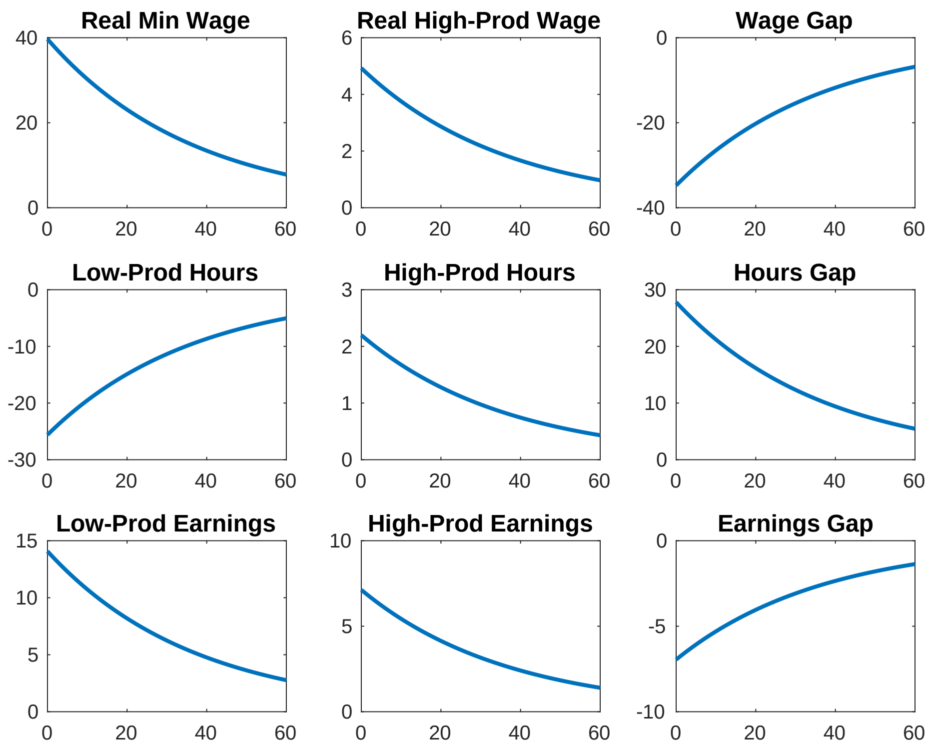

Figure 7: Distributional Effects: Expansionary Min Wage Hike

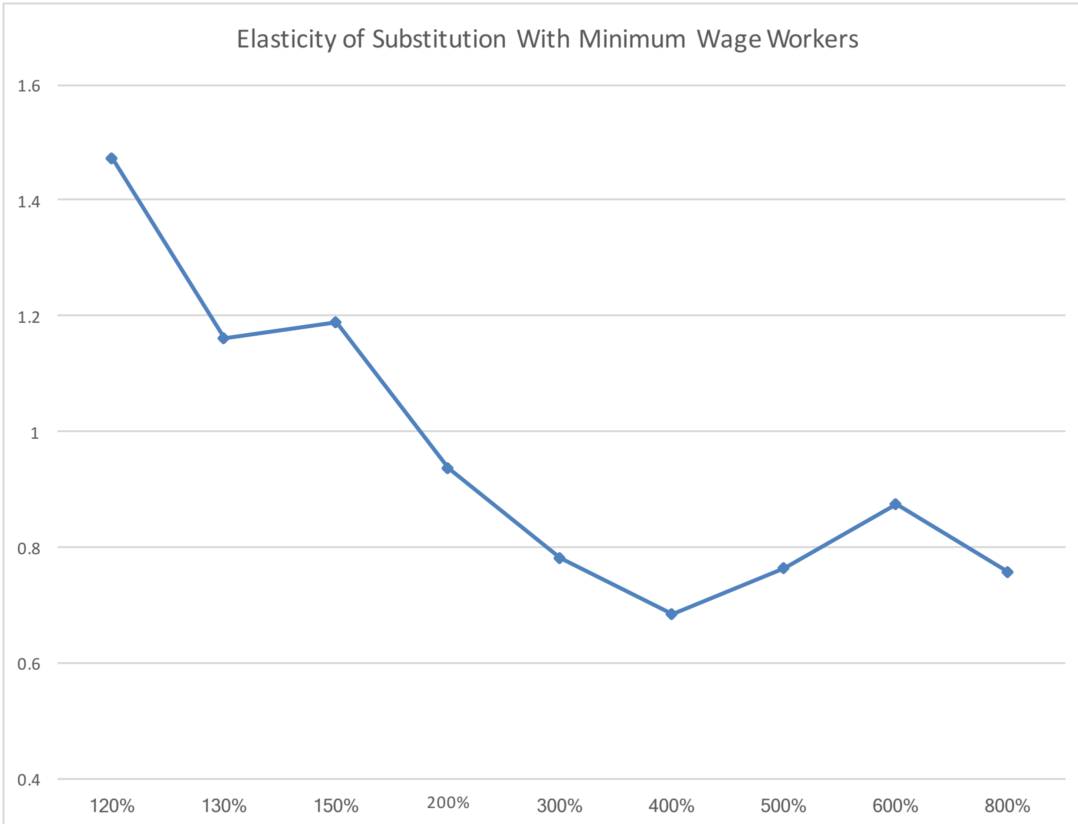

Figure 8: Elasticities Implied by Neumark, Schweitzer, & Wascher

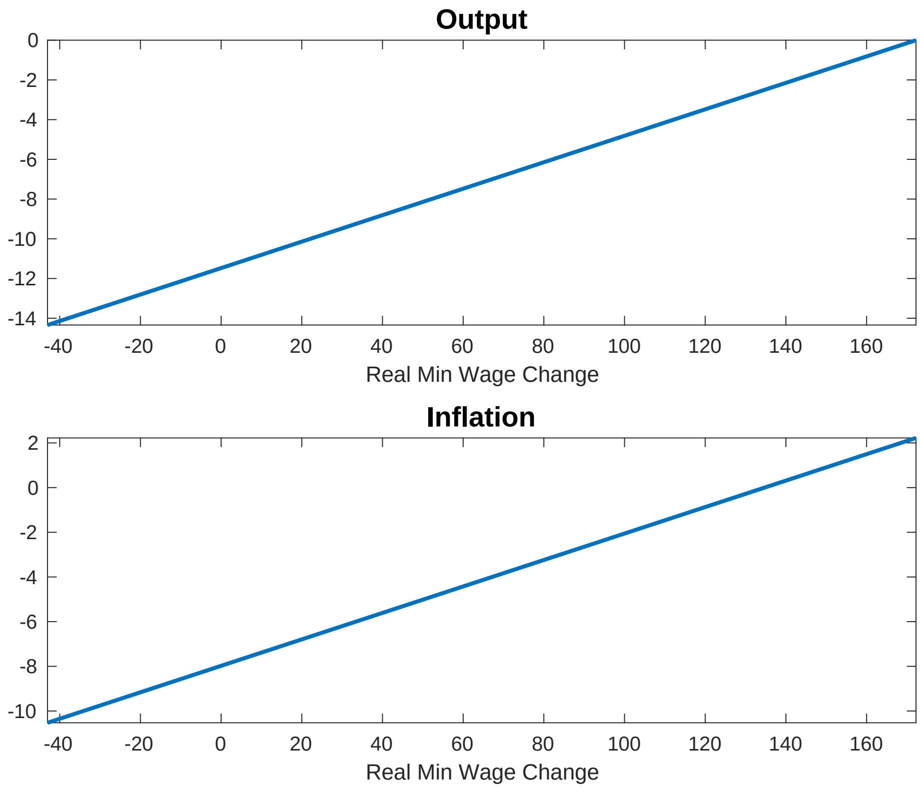

Figure 9: Effect of Indexing Min Wage During ZLB