Minimum Wages as Equilibrium Selection in an Expectations-Driven Liquidity Trap

Andrew Glover
University of Texas - Austin

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Abstract

A sufficiently high real minimum wage policy eliminates liquidity trap equilibria in the sticky-price New Keynesian model by putting a lower bound on deflation expectations. However, there is a tradeoff between economic stability and allocative efficiency, because the minimum wage shifts labor demand from low to high productivity workers. The minimum wage's effects are non-monotonic: a sufficiently high minimum wage eliminates the liquidity trap, whereas a low value actually makes it worse. Finally, higher real minimum wages are associated with higher inflation expectations in cross-sectional data, consistent with the theory.

Very Preliminary

1 Introduction

Many countries have undergone extended periods of near-zero nominal interest rates in recent years, which has challenged macroeconomists to explain how and when the zero lower bound (ZLB) will bind in the future, as well as what policies can stimulate the economy when it does. The first question is complicated because there are two common theoretical models of the zero lower bound in the New Keynesian literature. Models like Eggertsson and Woodford [3] posit a preference shock to aggregate demand is so large that the monetary authority lowers nominal rates to zero when attempting to offset it, while Benhabib, Schmitt-Grohé, and Uribe [1] show that New Keynesian models may have multiple equilibria, all but one of which converge to a liquidity trap in which the ZLB binds. Borağan, Cuba-Borda, and Schorfheide [2] provide evidence that both types of ZLB episodes have occurred in different countries, which is important because, as emphasized by Mertens and Ravn [5], the underlying cause of a ZLB episode dictates the efficacy of non-monetary policies aimed at alleviating the severe recession that it causes.

In this paper, I show how minimum wage regulations affect output and inflation in an expectations-driven liquidity trap, but more importantly, how such a sufficiently high real minimum wage can eliminate the liquidity trap altogether. The key mechanism is that the real unit cost of production must be low in a liquidity trap, as it must move in conjunction with output. Imposing a real minimum wage decouples the real unit cost of production from output, at least in deep recessions, thereby eliminating the rapid deflation required for pessimistic expectations of future demand to be self-fulfilling. Since previous work has shown that minimum wages also dampen output losses during a shock-driven ZLB episode (Glover [4]), I conclude that increasing the minimum wage can be expansionary during a ZLB episode of either type.

I first outline a continuous time New Keynesian model with heterogeneous labor inputs and minimum wage regulations. I show that this model may have multiple equilibria, one of which has high output and constant inflation but is unstable, while a continuum of equilibria that are Pareto inferior converge to an expectations-driven liquidity trap. Comparative statics demonstrate that a sufficiently high real minimum wage eliminates all but the best equilibrium, though it suffers in terms of allocative efficiency in most cases. I then test the ability of real minimum wages to affect inflation expectations in historical data and finally conclude.

2 Labor Market

Time is continuous and the economy comprises a continuum of producer-consumer households, indexed by $j \in [0,1]$.[1] Each household comprises one worker of two types, high-productivity and low-productivity, and produces a differentiated consumption good using the labor of other households' workers, which it sells to all other households under monopolistic competition and Rotemberg [8] price adjustment costs. Household j therefore chooses time paths of each consumption good, $(c_\ell)_{\ell\in[0,1]}$, bonds, b, labor demands, N_H and N_L, labor supplies, n_H and n_L, as well as prices and inflation rates, p_j and π, to maximize expected discounted utility:

$$\int_0^\infty e^{-\delta t}\left[\log\left[\int_0^1 c_{\ell,t}^{\frac{\epsilon-1}{\epsilon}}\,d\ell\right]^{\frac{\epsilon}{\epsilon-1}} - \sum_{i\in\{L,H\}}\Psi\frac{\nu}{1+\nu}n_i^{\frac{1+\nu}{\nu}} - 0.5\gamma\pi_t^2\right]dt, \tag{1}$$

subject to the constraints

$$\begin{aligned}
\dot{b}_t + \int_0^1 p_{\ell,t}c_{\ell,t}d\ell &= i_t b_t + W_{L,t}n_{L,t} + W_{H,t}n_{H,t} + Profit_{j,t} && (2)\\
Profit_{j,t} &= p_{j,t}y^d_{j,t}(p_{j,t}) - W_{L,t}N_{L,t} - W_{H,t}N_{H,t} && (3)\\
\dot{p}_{j,t} &= \pi_t p_{j,t} && (4)\\
N_{H,t}^{\alpha}\big(\zeta N_{L,t}\big)^{1-\alpha} &\geq y^d_{j,t}\big(p_{j,t}\big) && (5)\\
n_{H,t} &\leq \overline{n}_{H,t} && (6)\\
n_{L,t} &\leq \overline{n}_{L,t}, && (7)\\
& && (8)
\end{aligned}$$

where the labor supply constraints, $n_{i,t} \leq \overline{n}_{i,t}$, are taken as an arbitrary constraint by households, but will be equal to per-capita labor demand in equilibrium.

For the following analysis, note that the Rotemberg pricing assumption allows me to study a symmetric equilibrium with $p_{j,t} = P_t$ and $y^d_{j,t}(p_{j,t}) = Y_t$.

2.1 Labor Demand

I first characterize the labor demand decisions in production, which determine the real unit cost of production. The first-order conditions give the following expression for relative labor demands

$$\frac{N_{L,t}}{N_{H,t}} = \frac{1-\alpha}{\alpha}\frac{W_{H,t}}{W_{L,t}}. \tag{9}$$

[1]This formulation follows Michaillat and Saez [6].

Which yields high-productivity labor demand as a function of output and relative wages:

$$N_{H,t} = \left(\zeta \frac{1-\alpha}{\alpha} \frac{W_{H,t}}{W_{L,t}}\right)^{\alpha-1} Y_t. \tag{10}$$

These conditions hold in general, while the labor supply side of the economy depends on whether the minimum wage binds for either or both worker types. They imply that the real unit cost of production,

$$\frac{\mathcal{W}_t}{P_t} = \frac{W_{H,t}N_{H,t} + W_{L,t}N_{L,t}}{Y_t}, \tag{11}$$

which simplifies to[2]

$$\frac{\mathcal{W}_t}{P_t} = \alpha^{-1}\left(\zeta \frac{1-\alpha}{\alpha}\right)^{\alpha-1} W_{H,t}^{\alpha} W_{L,t}^{1-\alpha}. \tag{12}$$

2.2 Labor Supply

Under the assumption that $\alpha > 0.5$, there are three regions of labor supply outcomes that depend on aggregate output relative to the real minimum wage, ω. In the first case, output is sufficiently large that both $\frac{W_{L,t}}{P_t}$ and $\frac{W_{H,t}}{P_t}$ are above ω. In the second case, for intermediate output levels, $\frac{W_{L,t}}{P_t} = \omega < \frac{W_{H,t}}{P_t}$. Finally, for low levels of output, both real wages are below the minimum.

In general, the intratemporal Euler Equation for high-productivity workers holds for all but the lowest levels of output, in which case they receive the minimum wage. This gives a general expression for the high-productivity real wage as

$$\frac{W_{H,t}}{P_t} = \Psi^{\frac{\nu}{1-\alpha+\nu}} \left(\zeta \frac{1-\alpha}{\alpha}\right)^{\frac{\alpha-1}{1-\alpha+\nu}} Y_t^{\frac{1+\nu}{1-\alpha+\nu}} \left(\frac{W_{L,t}}{P_t}\right)^{\frac{1-\alpha}{1-\alpha+\nu}}. \tag{13}$$

For sufficiently high Y_t, the real wage of low-productivity workers is above the minimum, so their intratemporal Euler Equation is satisfied. Their real wage is then proportional to the high-productivity real wage:

$$\frac{W_{L,t}}{P_t} = \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{1+\nu}} \frac{W_{H,t}}{P_t}. \tag{14}$$

Equation (14) guarantees that low-productivity wages are lower than high-productivity wages whenever $\alpha > 0.5$. Using equation (10) to replace high-productivity labor and equation (14) to replace relative wages, the intratemporal Euler Equation for labor supply yields the following relationship between

[2]All derivations are in the appendix.

$\frac{W_{H,t}}{P_t}$ and output

$$\frac{W_{H,t}}{P_t} = \Psi\left[\zeta\left(\frac{\alpha}{1-\alpha}\right)^{-\frac{\nu}{1+\nu}}\right]^{\frac{\alpha-1}{\nu}} Y_t^{\frac{1+\nu}{\nu}}. \tag{15}$$

Importantly, the unit cost and both real wages are zero for $Y_t = 0$, which means that both will hit the real minimum wage for sufficiently low output. Furthermore, the low-productivity worker's wage will fall below the real minimum first. Therefore, for sufficiently high Y_t, equations (14) and (15) mean that the real unit cost of production from equation (12) is given by

$$\frac{\mathcal{W}_t}{P_t} = \Psi\alpha^{-1}\zeta^{\frac{\alpha-1}{\nu}}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} Y_t^{\frac{1+\nu}{\nu}}, \tag{16}$$

which is an increasing and convex function of output.

For sufficiently low Y_t, the low-productivity worker's wage in equation (14) falls below the real minimum wage ω, in which case the expression for the real wage of high-productivity workers also changes. Denote the level of output for which low-productivity workers earn the minimum wage by

$$\underline{Y}_L(\omega) = \left(\omega\mu^{-\frac{\alpha}{1+\nu}}\zeta^{\frac{1-\alpha}{\nu}}\Psi^{-1}\right)^{\frac{\nu}{1+\nu}}. \tag{17}$$

For $Y_t \leq \underline{Y}_L(\omega)$, the unit cost of production is found by replacing low-productivity real wages with ω in equation (13) and then evaluating equation (12). The unit cost in this region is still a convexly increasing function of output, but is flatter than the expression in equation (16). Finally, once output falls below

$$\underline{Y}_H(\omega) = \omega^{\frac{\nu}{1+\nu}}\left[\left(\zeta\frac{1-\alpha}{\alpha}\right)^{1-\alpha}\Psi^{-\nu}\right]^{\frac{1}{1+\nu}}, \tag{18}$$

all workers earn the real minimum wage, so the unit cost of production is just ω. The unit cost is therefore a piece wise continuous function in output and the real minimum wage that is constant for very low output, but strictly increasing for $Y_t \geq \underline{Y}_H(\omega)$ with a kink at $\underline{Y}_L(\omega)$.

3 Equilibrium System

The goods market is standard, so the intertemporal Euler Equation is given by

$$\frac{\dot{Y}_t}{Y_t} = i_t - \pi_t - \delta. \tag{19}$$

I will assume throughout that monetary policy sets nominal interest rates according to the Taylor Principle, so that $\phi > 1$ and the nominal rate is given by

$$i_t = \max\{\delta + \phi\pi_t, 0\}. \tag{20}$$

Pricing decisions, together with the real unit cost described in section 2, give the Phillips Curve

$$\dot{\pi}_t = \delta\pi_t - \kappa\frac{\mathcal{W}_t}{P_t} + \frac{\epsilon - 1}{\gamma}, \tag{21}$$

which I will plot in (Y, π), treating ω as a shifter (according to the characterization of $\frac{\mathcal{W}_t}{P_t}$ above).

3.1 Steady State Equilibria

As in well known, the standard New Keynesian model may have two steady-state equilibria (Mertens and Ravn, [5]). One of these equilibria features inflation on target (in this model zero) and high output. In the other, households expect a high interest rate, which leads them to have low demand, which causes deflation, and therefore a high real interest rate. This equilibrium is referred to as the "Expectations Driven Liquidity Trap".

Geometrically, these two equilibria correspond to two possible aggregate demand curves in (Y, π) space, as shown in Figure 1. The Phillips Curve without minimum wages is plotted as the upward sloping dashed line, while the horizontal dotted lines represent aggregate demand under the liquidity trap (lower line) and high output (upper line).[3] Note that the Phillips Curve has been drawn with an intercept below $-\delta$ so that it crosses both aggregate demand lines, which generates two steady state equilibria. The condition for this multiplicity to arise is simply

$$-\frac{\epsilon - 1}{\gamma} < -\delta, \tag{22}$$

which I will assume from here on.

Now consider the model with minimum wages. The aggregate demand lines are unchanged, but now the Phillips Curve is represented by the solid curve in Figure (1). There are now three sections of the Phillips Curve, corresponding to high output, intermediate levels of output, and low output. In the first region, the minimum wage is slack for all workers and the curve has the same slope as in the standard model. In the second region, the minimum wage

[3] I have drawn the Phillips Curve linearly for graphical simplicity. It is in fact convex for a finite value of ν, but the qualitative analysis is the same.

binds for just the low-productivity workers, and the curve is flatter than the standard model. Finally, for sufficiently low output, the minimum wage binds for all workers and the Phillips Curve is horizontal.

3.2 Comparative Statics

As can be seen in Figure (1), the effect of the real minimum wage depends on the levels of output where the slope changes. The figure illustrates the case when the real minimum wage is sufficiently low that the flat section of the Phillips Curve is below the liquidity-trap aggregate demand line, but where the Phillips Curve crosses the zero-inflation aggregate demand line at a level of output where low-productivity workers are paid a binding minimum wage. Figure (2) begins with this Phillips Curve and considers the effect of increasing the minimum wage to two higher values.

The first thing to note in Figure (2) is that any increase in the minimum wage reduces output in the zero-inflation steady state, as seen in the leftward movement of the points A_M and A_H. In each case, a higher minimum wage distorts the labor inputs away from low-productivity workers. The wage of high-productivity workers responds in equilibrium, but not fully, which reduces allocative efficiency and output.

While the level of output associated with zero inflation falls monotonically with the real minimum wage, the effect on output in the liquidity-trap steady state is more complex. Starting from a low minimum wage, a small increase in the minimum wage from ω_L to ω_M moves the Phillips Curve from PC_L to PC_M. This reduces steady-state output in the liquidity trap from Z_L to Z_M, mirroring the reduction in zero-inflation output. As in the zero-inflation steady state, this reduction in output is due to a reduction in allocative efficiency.

When the real minimum wage is further increased to ω_H, the Phillips Curve moves further upward to PC_H. In this case, the flat region is above the liquidity-trap aggregate demand line and there is no longer a steady state with low output and deflation. The liquidity trap requires that households' expectation of a high real interest rate due to deflation is realized because deflation actually occurs in the face of low aggregate demand. Rapid deflation is only optimal if real wages fall with output, which is not possible in the face of a high real minimum wage.

A sufficiently high real minimum wage therefore eliminates the liquidity-trap steady-state equilibrium altogether. Furthermore, the liquidity-trap steady state is the limit of a continuum of dynamic equilibrium paths, all of which are also eliminated when the real minimum wage is set to ω_H. The only equilibrium is therefore the zero-inflation steady state, which presents the tradeoff from having a high real minimum wage since it features a lower level of output than when the minimum wage is low, but is now stable.

The tradeoff between allocative efficiency and stability relies on the minimum wage binding for low-productivity workers for all levels of output up to the zero-inflation steady state. If the low-productivity real wage is above the minimum at the zero-inflation steady state, then it is possible to eliminate the liquidity-trap steady state (and all equilibria that converge to it) without distorting labor inputs in the zero-inflation steady state. This situation is shown in figure (3). Even in this case, raising the minimum wage may reduce output in the liquidity-trap steady state, which can be seen by the movement from point Z_L to Z_M.

4 Empirical Test

The minimum wage eliminates the expectations-driven liquidity trap by altering inflation expectations. A natural question is whether the minimum wage empirically affect inflation expectations, which I test using cross-sectional data from the Michigan Survey of Consumers.

4.1 Data Description and Summary Statistics

I make use of three data sources. The first is a panel of state-level nominal minimum wages at a monthly frequency, constructed and made public by Neumark [7]. This is aggregated to census region and made real using regional consumer price indices. Figure (4) plots each region's real-minimum wage from January 1978 to December 2017.

These regional real minimum wage series are then merged with the Michigan Survey of Consumers (MSC). This data set provides monthly repeated cross sections of individual expectations of inflation over the next twelve months, as well as basic demographic and socioeconomic status information of respondents. Importantly, it also reports the census region (one of "Northeast, South, North Central, or West") in which the respondent resides.

Inflation expectations are elicited at the interviewee level in the MSC on a monthly basis, starting in January 1978. An interviewee is first asked "During the next 12 months, do you think that prices in general will go up, or go down, or stay where they are now?" If they say that prices will go up or down, then they are asked "By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?", with further probing for extreme answers (more than $+/-5\%$) and verification that signs are correct

Table 1: Inflation Expectations and Real Min. Wage

	$\mathbb{E}_{i,r,t}\pi_{t+1} = \beta w_{r,t} + \mu_t + \gamma_r + G_r(X_{i,r,t}) + \varepsilon_{i,r,t}$				
	(1)	(2)	(3)	(4)	(5)
Real Min Wage	0.034***	0.033**	0.034**	0.028**	0.028**
	(0.006)	(0.006)	(0.007)	(0.005)	(0.005)
Education	N	Y	Y	Y	Y
Age	N	N	Y	Y	Y
Gas Expectation	N	N	N	Y	Y
Real Income	N	N	N	N	Y
Obs.	246,600	244,005	243,015	112,423	107,077
R^2	0.107	0.111	0.113	0.053	0.060

Notes: Estimated regression in header of table. Dependent variable is individual-level inflation expectations from the Michigan Survey of Consumers. Independent variable of interest is the logarithm of regional real minimum wage, multiplied by 100. Standard errors clustered by region and month in parenthesis, significance levels reported at 10%(*), 5%(**) and 1%(***). Controls are all interacted with region. "Education" and "Age" have fixed effect for each year of education and age reported. "Gas Expectation" has fixed effect for each decline of expected gas price growth over the next year. "Real Income" enters regression linearly.

(i.e. a person who says they expect prices to go up but then answers −5% is recoded as responding with 5%). Further probes are made for people who say that they believe prices will stay the same (to verify that they didn't intend that *inflation* will remain the same). Figure (5) plots the resulting mean and median inflations rates for each region.

4.2 Regression Results

Table 1 reports the estimated response of inflation expectations to the regional real minimum wage from the following regression

$$\mathbb{E}_{i,r,t}\pi_{t+1} = \beta w_{r,t} + \mu_t + \gamma_r + G_r(X_{i,r,t}) + \varepsilon_{i,r,t}, \tag{23}$$

where $\mathbb{E}_{i,r,t}\pi_{t+1}$ is the expected inflation over the next year for person i in region r at date t. The coefficient of interest is β, which measures the response of inflation expectations to a 1% increase in the real minimum wage of region r at date t.

All specifications include aggregate time fixed effects for two reasons. First, inflation expectations have trended downward nationally and the real minimum wage has a U-shape, so a positive relationship in the time series may be spurious. Second, the theory requires a higher minimum wage to increase inflation expectations at a liquidity trap, when there is no response in the nominal interest rate through monetary policy. This setting is approximated in the panel regression by assuming that any nominal interest rate response to common changes in the minimum wage is contained in the time fixed effect. In addition, each specification includes region fixed effects since the regional price indices are constructed with different average price levels in the base year. The remaining controls are included iteratively and are interacted with region dummies. Controls include real household income as well as fixed effects for years of education, age, and decile of expected gas price growth over the next year.

The regression results are summarized in Table 1, where columns (1) - (5) vary only by which controls are included. The point estimate for the real minimum wage's effect on inflation expectations is always positive at the 5% level, suggesting that higher minimum wages can increase inflation expectations. The coefficients are similar across specifications, ranging from 0.028 to 0.034, which implies that a 10% increase in the real minimum wage is associated with approximately 0.3% higher inflation expectations, on average. Since that the average inflation expectation in the Michigan Survey of Consumers is 4.3% overall and 3.1% since 1985, this represents a substantial increase.

5 Conclusion

I have extended the sticky-price New Keynesian model to include heterogeneous labor and minimum wage policy. This slightly richer model opens the door to a new macroeconomic role of minimum wage regulation. By decoupling wages and output in a deep recession, deflation is bounded and a continuum of equilibrium featuring self-fulfilling bouts of pessimism are eliminated. On the other hand, the remaining equilibrium exhibits a distortion between high and low productivity labor in most cases, creating a trade off between aggregate stability and allocative efficiency.

References

[1] J. Benhabib, S. Schmitt-Grohé, and M. Uribe. The perils of taylor rules. *Journal of Economic Theory*, 96(1-2):40–69, 2001.

[2] S. Borağan Aruoba, P. Cuba-Borda, and F. Schorfheide. Macroeconomic dynamics near the zlb: A tale of two countries. *The Review of Economic Studies*, 85(1):87–118, 2017.

[3] G. B. Eggertsson and M. Woodford. Policy options in a liquidity trap. *The American Economic Review*, 94(2):76–79, 2004.

[4] A. Glover. Aggregate effects of minimum wage regulation at the zero lower bound. *Journal of Monetary Economics*, Forthcoming.

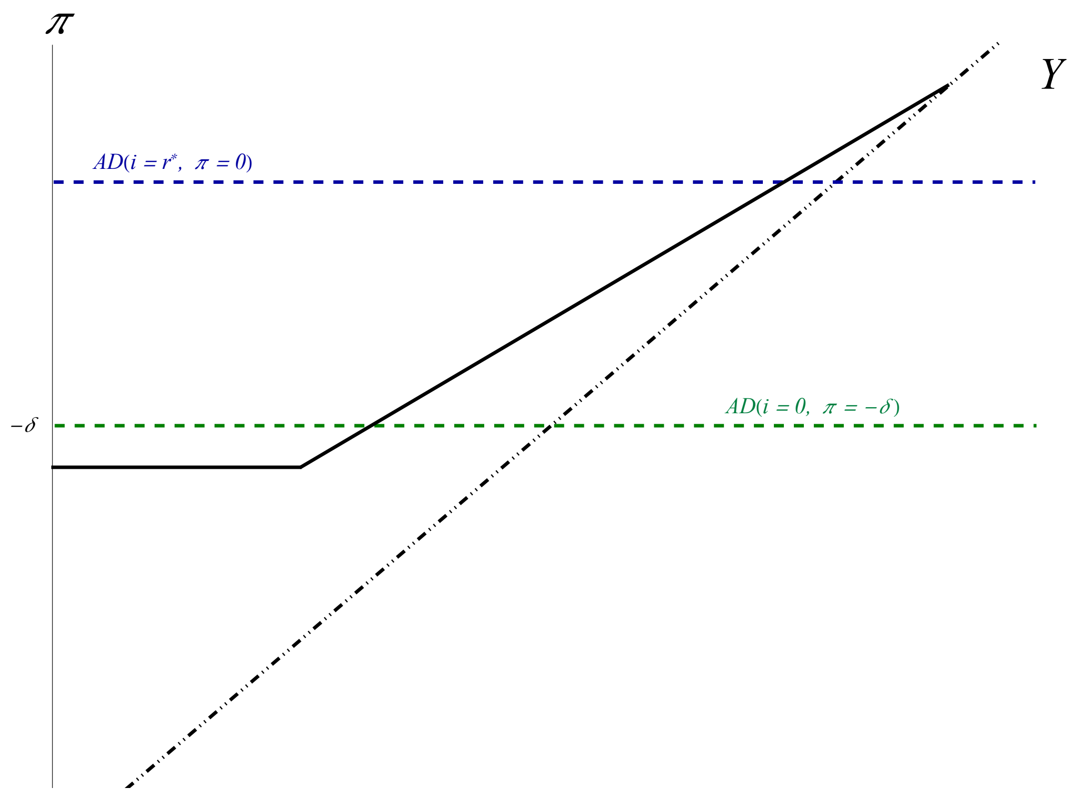
[5] K. R. Mertens and M. O. Ravn. Fiscal policy in an expectations-driven liquidity trap. *The Review of Economic Studies*, 81(4):1637–1667, 2014.

[6] P. Michaillat and E. Saez. A new keynesian model with wealth in the utility function. *Manuscript*, page 34, 2018.

[7] D. Neumark. Minimum wage and living wage data set. =https://www.socsci.uci.edu/ dneumark/datasets.html, 2018.

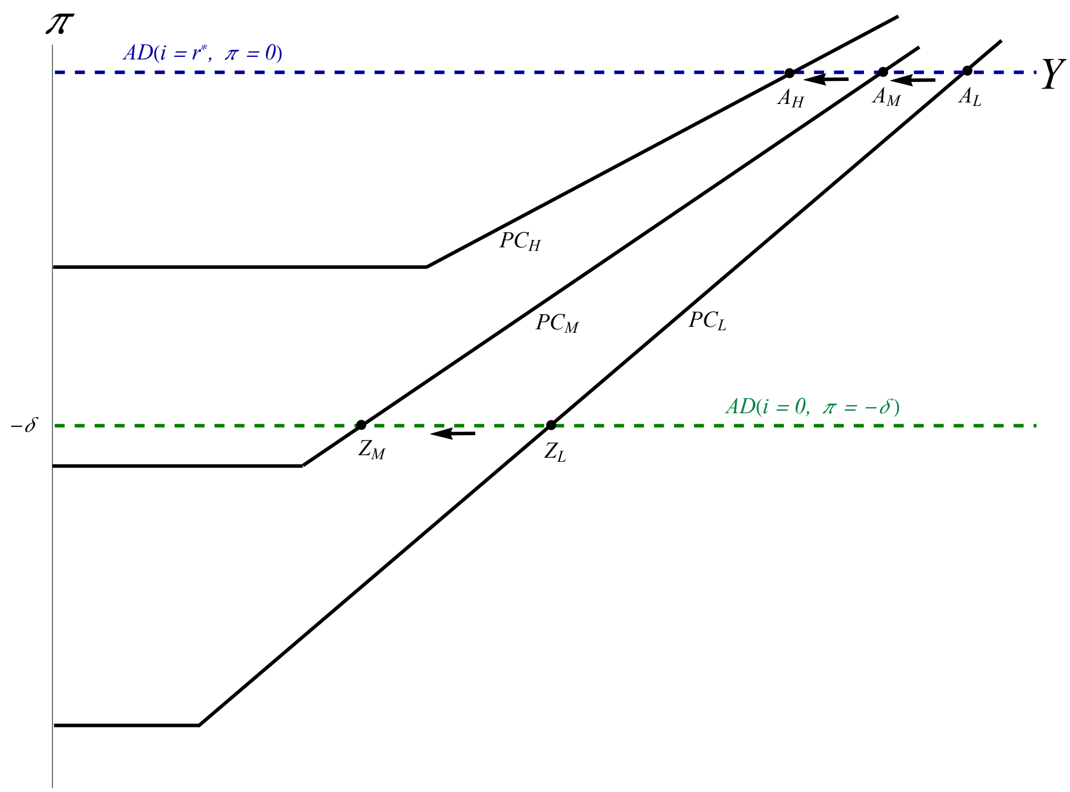
[8] J. J. Rotemberg. Sticky prices in the united states. *Journal of Political Economy*, 90(6):1187–1211, 1982.

Figure 1: Equilibrium Determination



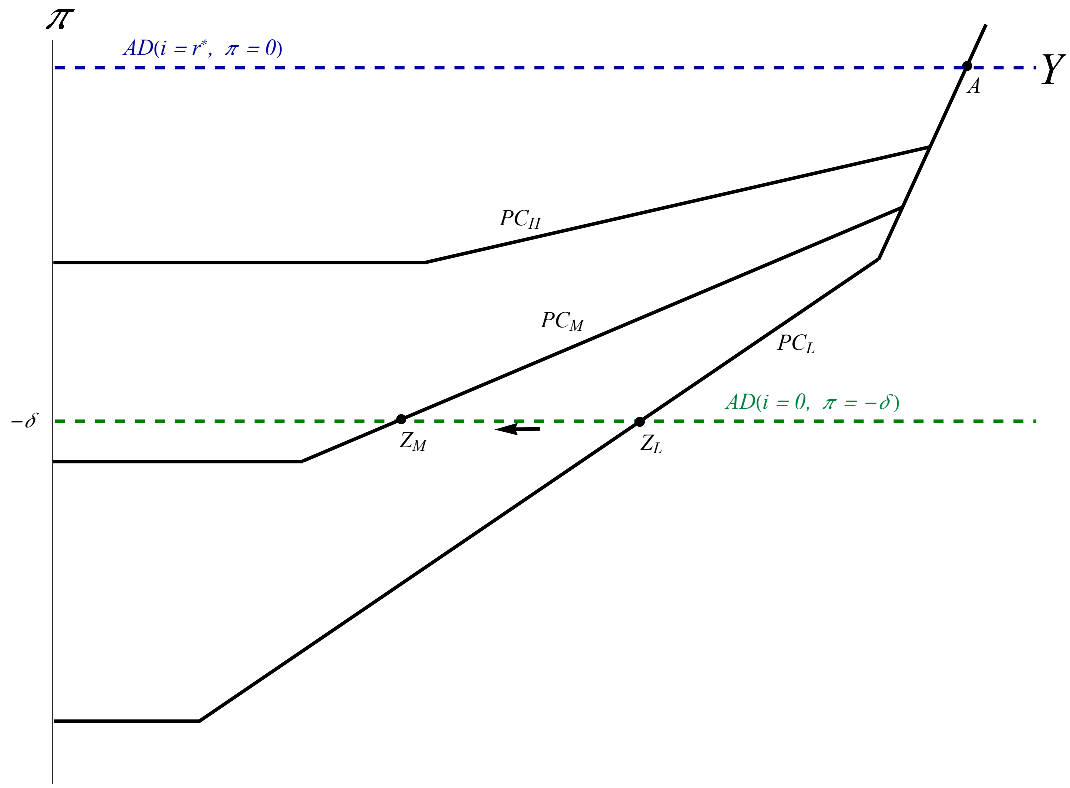
Notes: Dashed line shows Phillips Curve locus for $\dot{\pi}_t = 0$ in model without minimum wages. Solid line shows Phillips Curve locus for $\dot{\pi}_t = 0$ in model with a positive real minimum wage. Dotted lines correspond to $\dot{Y}_t = 0$ loci under two steady-state inflation rates: $\pi = -r^*$ and $\pi = 0$.

Figure 2: Equilibrium Comparative Statics



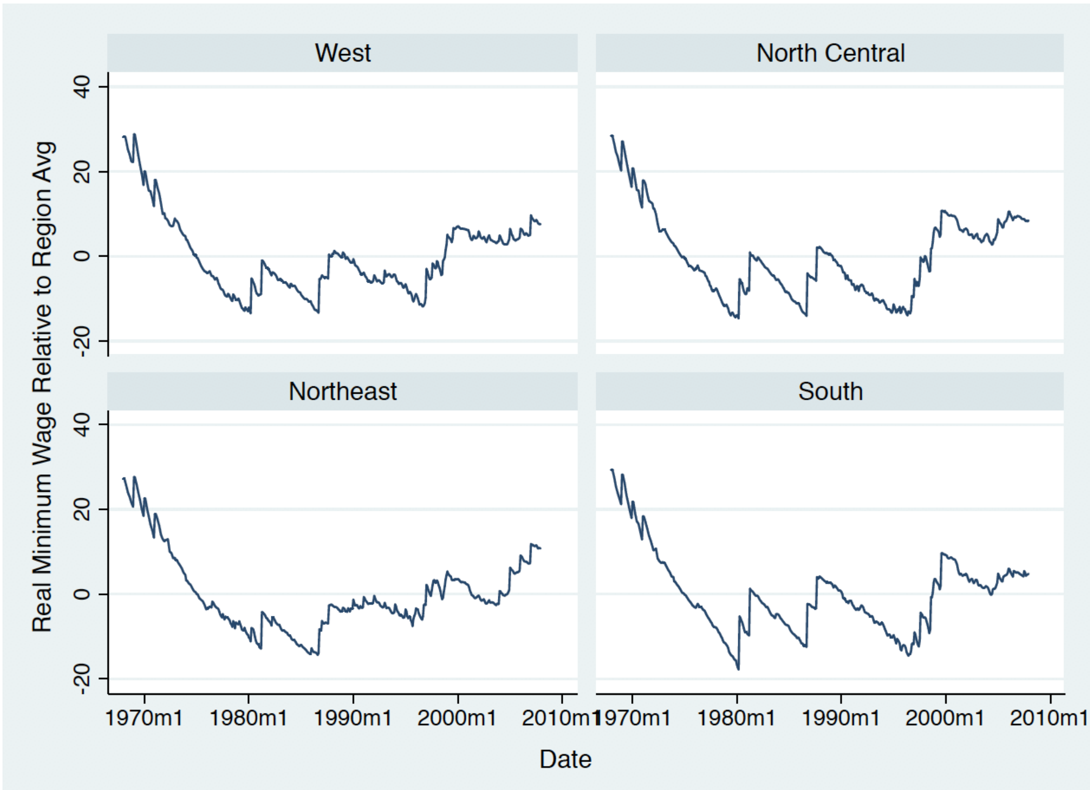
Notes: Solid lines show Phillips Curve loci for $\dot{\pi}_t = 0$ in model with different real wages, $\omega_L < \omega_M < \omega_H$. Dotted lines correspond to $\dot{Y}_t = 0$ loci under two steady-state inflation rates: $\pi = -r^*$ and $\pi = 0$.

Figure 3: Equilibrium Comparative Statics



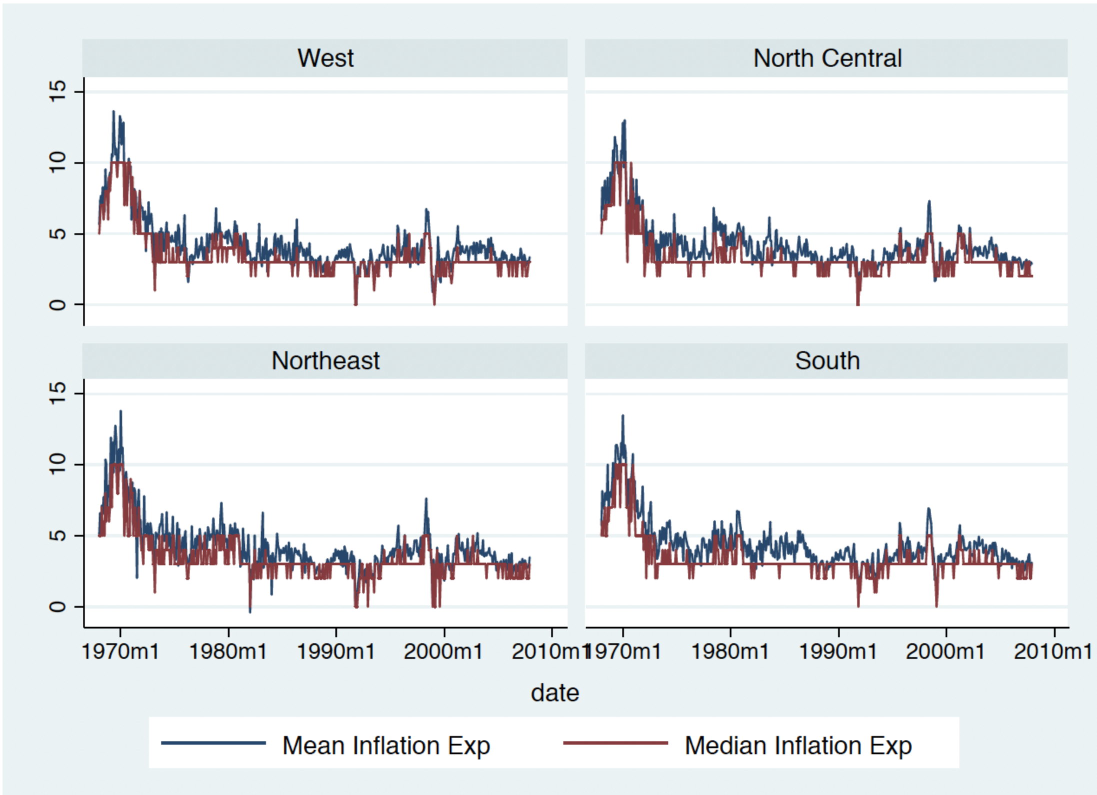
Notes: Solid lines show Phillips Curve loci for $\dot{\pi}_t = 0$ in model with different real wages, $\omega_L < \omega_M < \omega_H$. Dotted lines correspond to $\dot{Y}_t = 0$ loci under two steady-state inflation rates: $\pi = -r^*$ and $\pi = 0$.

Figure 4: Real Minimum Wages by Region



Notes: Plots show average of nominal minimum wage for states in each census region, deflated by regional CPI, logarithmically transformed. All series are relative to the region's time series mean.

Figure 5: Inflation Expectations by Region



Notes: Plots show average and median of household inflation expectations in each census region.