Can Capital Deepening Explain the Global Decline in Labor’s Share?*

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Abstract
We estimate that a unitary aggregate elasticity of substitution between capital and labor is economically and statistically consistent with cross-country data – capital deepening cannot explain the global decline in labor’s share. Our methodology derives from inter-steady-state transitions in the Neo-Classical growth model. The elasticity of substitution is identified from the correlation between trends in labor’s share, investment prices, and consumption growth across countries. We show that previous estimates of this elasticity from international data are biased upwards because they omitted a theoretically and empirically important term related to consumption growth.

Keywords: Elasticity of Substitution, Labor Share, Investment Prices
JEL Codes: E21, E22, E25

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1 Introduction

Historically, Kaldor’s [8] balanced growth facts argued for a unitary elasticity of substitution between capital and labor due to the near constancy of labor’s share of income. As documented by Karabarbounis and Neiman [9], this near constancy is no more – labor’s share has declined globally since the 1980s. This downward trend, which has occurred along with rising income and wealth inequality (Piketty [14], Piketty and Zucman [15]), has renewed macroeconomists’ interest in estimating the aggregate elasticity of substitution between capital and labor (henceforth referenced as $\sigma \geq 0$). This parameter is pivotal: if it is smaller (greater) than one then anything that causes an increase in the capital-labor ratio will increase (reduce) labor’s share. Specifically, if it is significantly larger than one then the global decline in labor’s share can be explained by capital deepening due to falling investment prices. Using a large cross section of countries, we estimate that $\sigma \approx 1$ and conclude that capital deepening cannot explain the global decline in labor’s share.

We extend the cross-country estimation strategy of Karabarbounis and Neiman (henceforth KN) to identify $\sigma$ from the capital demand function of a profit-maximizing firm in the Neo-Classical growth model. This demand function implies that a 1% fall in the rental rate of capital should reduce labor’s share of income by $(\sigma - 1)\%$. The idea of the cross-country estimation strategy is to correlate country-specific trends in rental rates and labor’s share to estimate $\sigma$. Unfortunately, rental rates are not readily available for a large cross section of countries, so we use the inter-temporal Euler Equation for investment to find an appropriate proxy. This condition implies that the rental rate depends on the the relative price of investment goods and consumption growth. We estimate $\sigma$ from various data sources, cross-sectional samples of countries, and statistical models. Our estimates are typically near to and statistically indistinguishable from one. The average point estimate from our baseline model is $\hat{\sigma} = 1.018$, suggesting that the observed decline in world-wide investment prices can explain only 4.4% of the observed decline in labor’s share.

These estimates stand in sharp contrast to those of KN, who estimate a large value of $\sigma$ and conclude that over half of the global decline in labor’s share can be explained by declining investment prices. The most economically important difference in our approaches is that KN’s estimating equation for $\sigma$ omits consumption growth and instead uses investment prices alone to proxy for the rental rate. This proxy is valid for observations beginning and ending in steady states, while our proxy is valid both in steady
state (in which case it is identical to their’s) and along transition paths. Omitting consumption growth for countries along transition paths can cause substantial upward bias when the true value of $\sigma$ is above one.\(^1\) We show this in detail in Section 5.1, but the intuition can be seen from a simple example. Suppose that $\sigma$ is slightly above one and the relative price of investment followed a path of geometric decline, eventually falling by 1%.\(^2\) Households choose a smooth transition path of consumption to the new (higher) steady-state, along which consumption growth is declining. The path of consumption implies a slow rise in the investment rate and gradual capital deepening. Now consider an econometrician who’s data started late in the transition: she would observe a flat path for the investment price but a downward trend in labor’s share. If she ignored the trend in consumption growth then she would erroneously infer a large value of $\sigma$.

The above logic aligns with our empirical results: we estimate larger $\sigma$ from a mis-specified model that omits consumption growth. There is no reason to omit the consumption growth term in principle, but in practice the exact specification requires us to assign additional values to technological and preference parameters. We may therefore prefer to estimate the misspecified model if the upward bias of $\hat{\sigma}$ is expected to be low. We therefore use the exact structural model as our baseline, but also present results from the misspecified model, which omits data on consumption growth.

When we estimate the misspecified model, we find that $\hat{\sigma}$ is sensitive to the subsample of countries used in the estimation. This is important because the econometrician determines this subsample by excluding countries with time series for investment prices, labor’s share, and consumption shorter than some threshold $T_{\text{min}}$. We find that $\hat{\sigma}$ from the misspecified model is largest for KN’s preferred $T_{\text{min}} = 15$, but that the estimate is just as often below one as above as we vary $T_{\text{min}}$ and is only significantly above one for one-third of subsamples.

The cross-country approach we employ is one of three broad empirical strategies used to identify the aggregate elasticity of substitution. One alternative, pursued by Antràs [2], uses aggregate time series variation in the United States and estimates $\sigma \leq 1$. There is also a large literature on estimating the elasticity of substitution from micro data. A particularly relevant example is Oberfield and Raval [12], who estimate these micro elasticities and aggregate them to compute a $\sigma$ substantially below one. This paper

\(^1\)The direction of bias depends on the sign of $\sigma - 1$ and the correlation between the consumption growth term and investment prices. On average, our estimates of $\sigma$ are slightly above one and the consumption growth term is both theoretically and empirically positively correlated with investment prices.

\(^2\)For example, imagine the investment price falls by half of the remaining distance each year.
therefore reconciles the cross-country estimates with these and other previous studies, which typically estimate $\sigma$ below one.\footnote{Chirinko [5] and Leon-Ledesma, et al [11] provide surveys of the literature on estimating the elasticity of substitution.} Although our point estimates are larger than those from these previous studies, the economic implication is identical: the global decline in labor’s share cannot be explained by capital deepening.

Although our estimates of $\sigma$ indicate that capital deepening has not caused the global decline in labor’s share documented by Karabarbounis and Neiman, the fact that it has fallen so broadly demands explanation. The literature is rich in potential explanations. Some of the decline may be due to mismeasurement (see Rognlie [16] for a discussion of housing’s effect on capital’s share, Bridgman [4] on the importance of measuring depreciation and production taxes, and Elsby, et al [6] for a discussion of proprietor’s income). Technology and capital accumulation could play a more subtle role (see Koh, et al [10] on intellectual property (IP), Alvarez-Cuadrado, et al [1] on sectoral shifts towards services, and Orak [13] on the elasticity of substitution between equipment and routine-task labor). Finally, there may be a role for changes in labor market competitiveness (see Autor, et al [3] on the rising concentration of market power of “superstar firms”, Glover and Short [7] on the effect of demographics when older workers have little bargaining power, and Elsby, et al [6] on the effect of import competition at the industry level).

We proceed by outlining the structural theory that relates labor’s share to investment prices and the economic importance of $\sigma \approx 1$ in that framework. We derive the appropriate reduced form model to estimate $\sigma$, which we do using various data sources. We then discuss the bias from omitting consumption growth and capital augmenting technological progress, compare our estimates to the literature, and conclude.

\section{Labor’s Share in Theory}

Our framework to estimate $\sigma$ is based on transition paths in the Neo-Classical growth model. The relative price of investment goods in country $i$ follows an exogenous and deterministic path to a new steady state, $P^i$. Starting at an initial $P^i_0$, which may or may not be a steady-state, the path is written as:

$$\log P^i_t = \log P^i + \log \xi^i_t. \quad (1)$$
Where \( P_i \) is the eventual steady-state investment price and \( \log \xi_i^t \) is a sequence that tends to zero (i.e. \( \log \xi_i^t \) describes the transition path of investment prices). We think of the data as representing, for each country, a snapshot of this transition path.\(^4\)

We assume that goods and factor markets are competitive.\(^5\) Each country is populated by a large number of representative households, each of whom chooses sequences of consumption \( c_t \), labor \( n_t \), bonds \( b_t \), and investment \( x_t \) to solve the problem:

\[
\max_{(c_t, n_t, x_t, k_{t+1}, b_{t+1})} \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t)
\]

subject to:

\[
c_t + P_i^t x_t + Q_i^t b_{t+1} = R_i^t k_t + W_i^t n_t + b_t
\]

\[
k_{t+1} = (1 - \delta) k_t + x_t
\]

\( k_0, b_0 \) given.\(^5\)

We denote the price of a risk-free bond as \( Q_i^t \), the real wage by \( W_i^t \), and the rental rate for capital by \( R_i^t \). We are intentionally agnostic about the determination of \( Q_i^t \) and \( P_i^t \). For \( Q_i^t \), our estimation approach remains valid whether the bond market is closed and \( Q_i^t \) is determined in equilibrium or if each country is a small open economy and takes \( Q_i^t \) exogenously. Likewise, it does not matter if the price of investment goods changes because each country has idiosyncratic trends in their production technology for investment goods or if they face exogenous changes in the price of investment goods on the international market.

We will make use of the inter-temporal Euler Equations from this problem:

\[
Q_i^{t-1} = \frac{R_i^{t+1} + (1 - \delta) P_i^{t+1}}{P_i^t}
\]

\[
Q_i^{t-1} = \frac{u_c(c_i^t, 1 - n_i^t)}{\beta u_c(c_{i+1}^t, 1 - n_{i+1}^t)}.
\]

The first is a no-arbitrage condition between the gross rates of return on bonds and investment and the second relates growth in marginal utility to the return on bonds.

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\(^4\)These equations hold even if \( \log \xi_i^t \) does not converge to zero and our baseline empirical model remains valid. We assume that that \( \log \xi_i^t \) converges to zero for comparability to KN in Section 5.2.

\(^5\)Allowing for goods market markups is straightforward in theory, but introduces capital’s share and profit’s share of income as distinct variables. These cannot be separated in the data, so require imputation. In Appendix 1, we estimate \( \sigma \) using KN’s markup imputation procedure. The estimates are similar to our baseline under perfect competition, but are subject to (potentially large) measurement error, as we show in the appendix.
We again emphasize that these equations hold independently of how \( Q^i_t \) and \( P^n_t \) are determined. Combining these two equations yields a relationship between the rental rate, investment prices, and growth in marginal utilities:

\[
R^i_{t+1} = P^n_t \left[ \frac{u_c(c^i_t, 1 - n^i_t)}{\beta u_c(c^i_{t+1}, 1 - n^i_{t+1})} \cdot \frac{P^i_{t+1}}{P^n_t} + \delta - 1 \right].
\] (8)

A representative firm rents capital and hires labor to produce consumption goods using a constant returns to scale production function. We adopt the constant elasticity of substitution functional form:

\[
Y_t^i = \left[ \frac{\alpha^i_k (A^i_t K^i_t)^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha^i_k) (B^i_t N^i_t)^{\frac{\sigma - 1}{\sigma}}}{\frac{\sigma - 1}{\sigma}} \right]^\frac{\sigma}{\sigma - 1},
\] (9)

where \( \sigma \) is the elasticity of substitution between capital and labor and is the parameter of interest in this paper. The firm’s capital demand equation is the basis for our empirical model and is given by:

\[
R^i_t = \alpha^i_k (A^i_t) \frac{\sigma}{\sigma - 1} \left( \frac{Y_t^i}{K_t^i} \right)^\frac{1}{\sigma}.
\] (10)

Denoting labor’s share of income as \( s^i_t \equiv 1 - \frac{R_t K_t}{Y_t} \), the theoretical relationship between rental rates and labor’s share can now be written as:

\[
1 - s^i_t = (\alpha^i_k)^{\sigma} \left( \frac{A^i_t}{R^i_t} \right)^{\frac{\sigma - 1}{\sigma}}.
\] (11)

All else equal, a rise in the rental rate of capital will cause labor’s share to rise (fall) when \( \sigma \) is greater (smaller) than one. If we had data on the rental rate, \( R^i_t \), then we could use Equation (11) to estimate \( \sigma \). This data is not readily available, however, so we use Equation (8) to proxy for the rental rate.

The idea of using Equation (8) to proxy for the rental rate is shared with KN, but we differ from them because we use the entire right-hand side of that equation whereas they use only the investment price term. If we have data points that we know correspond to steady-states, then these two proxies are equivalent since Equation (8) says that the rental rate is exactly proportional to the price of investment goods in steady-state (assuming constant structural parameters \( \beta \) and \( \delta \)). Away from steady state (for example, along a transition path between steady states) the rental rate depends on both the investment price and consumption growth.

By combining Equations (8) and (11) we can relate labor’s share, investment prices,
and consumption as:

\[ \Delta \log(1 - s^i_t) = (\sigma - 1) \left[ \Delta \log A^i_t - \Delta \log P^i_t - \Delta \log \zeta^i_t \right], \]

(12)

where \( \zeta^i_t \) is the consumption growth term:

\[ \zeta^i_t = \frac{u_c(C^i_{t-1}, 1 - N^i_{t-1}) P^i_{t-1}}{\beta u_c(C^i_t, 1 - N^i_t) P^i_t} + \delta - 1. \]

(13)

Finally, we follow KN by approximating the left-hand side as:

\[ \Delta \log(1 - s^i_t) \approx -s^i_1 - s^i_i \Delta \log s^i_t, \]

(14)

where \( \bar{x}^i \) is the average of variable \( x \) for country \( i \) over the years for which it is observed. This gives Equation (15), which will be the theoretical basis for our baseline estimating equation in Section 5.

\[ \frac{s^i_i}{1 - s^i_i} \Delta \log s^i_t = (1 - \sigma) \left[ \Delta \log A^i_t - \Delta \log P^i_t - \Delta \log \zeta^i_t \right]. \]

(15)

3 Economic Importance of \( \sigma \approx 1 \)

Our estimates of \( \sigma \) are often statistically indistinguishable from one, but it is useful to quantify the economic importance of \( \sigma \) slightly above one. We consider four values of \( \sigma \) and the model’s prediction for labor’s share when faced with empirical trends in investment prices. Figure 1 plots the predicted labor’s share series for \( \sigma \in \{1, 1.018, 1.134, 1.288\} \). These values correspond to Cobb-Douglas, the average of our baseline estimates, the average \( \hat{\sigma} \) from KN’s subsample of countries, and the average of KN’s point estimates. Table 1 contains the cumulative change in predicted global labor’s share for each of these elasticities.

The predicted change in labor’s share when \( \hat{\sigma} = 1.018 \) is nearly equivalent to Cobb-Douglas – the fall in labor’s share is only 0.30 percentage points, which is 4.4% of the global decline in labor’s share. If we instead use our estimate from the subsample with \( T_{\text{min}} = 15, \ \hat{\sigma} = 1.134 \), then the decline is more substantial at 2.3 percentage points, although even this accounts for only one third of the observed decline in labor’s share.

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6This approximation does not significantly change our estimates of \( \sigma \). We use this approximation so that we replicate KN’s estimates precisely in Section 5.2.
This highlights the differences between our estimates and those of KN: using the average of the point estimates from their specification, $\hat{\sigma} = 1.288$, changes in investment prices predict a substantially larger decline of global labor’s share, which is seen in the grey dotted line in Figure 1. The predicted decline in labor’s share is 3.6 percentage points, which amounts to 52.7% of the observed decline.

Whether a large share of the global decline in labor’s share can be explained by capital deepening due to declining investment prices therefore rests on whether $\sigma$ is substantially greater than one. We now estimate $\sigma$ using various series, samples, and empirical models. The vast majority of estimates indicate that $\sigma \approx 1$.

4 Data

Our data set is an unbalanced panel comprising 104 countries with data available for some years between 1975 and 2010. The panel includes data on labor’s share, investment prices, and aggregate consumption. We will focus on medium to long-run trends, rather than estimate $\sigma$ from year-to-year variation.7 We first obtain measures, for each country, of the medium to long-run trends in the time series of each variable (we refer to this as the “first stage”). We then estimate $\sigma$ in Equation (15) from the cross section of trends constructed in the first stage (we refer to this as the “second stage”).

4.1 Description of Variables

We first describe the series for the first stage and then describe the measure of the long-run trends used in the second stage. For the first stage, we need time series for the relative price of investment goods, labor’s share, and consumption for each country.

For the relative price of investment goods, we present results from two publicly available series: the Penn World Table (PWT) and the World Bank’s World Development Indicators (WDI). For labor’s share of income we consider two measures: a hybrid measure which uses corporate-sector labor’s share whenever available and an economy-wide measure of labor’s share when the corporate-sector is missing, and the subsample using only labor’s share of the corporate sector.8 The hybrid measure uses corporate

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7Our data construction and focus on medium to long-run trends follow KN. For all samples and specifications, point estimates using year-to-year changes are consistently near one and never statistically different.

8The series for labor’s share and investment prices were created and shared by KN. The labor’s share series include collected time series from country specific sources, as well as series from OECD and UN.
labor’s share when available because it is cleaner: it avoids the difficulty of allocating proprietor’s income between labor and capital. The hybrid series is our baseline in order to maximize sample size in the second-stage, although we get nearly identical results when estimating on the smaller sample with corporate labor’s share alone.

For consumption data, we use real per-capita consumption series from the Penn World Table. In order to construct the time series for $\zeta^i_t$, we begin by assuming a separable functional form for period utility and a constant coefficient of relative risk aversion in consumption: $u(c_t, 1 - N_t) = \frac{c_t^{1-\theta}}{1-\theta} + \nu(1 - N_t)$. Our baseline further restricts these preferences to be consistent with balanced growth, so that $\theta = 1$ and $u(c_t, 1 - N_t) = c_t^{-1}$. Finally, we set $\delta = 0.10$ and $\beta = 0.91$. For country-year observations where consumption growth is sufficiently small, $\zeta^i_t$ measured by equation (13) may be negative and $\log \zeta^i_t$ undefined. Country-year observations for which $\log \zeta^i_t$ is undefined are dropped.

Since we must drop a small share of country-years for which $\zeta^i_t$ is undefined, we also provide estimates using the ex post real interest rate as a proxy for $\frac{U_c(c_t, \ell_t)}{\beta U_c(c_t+1, \ell_{t+1})}$. Data on the real interest rate comes from the World Bank’s World Development indicators, and is measured as the deposit rate less the growth rate of the GDP deflator. The real interest rate has the advantage of being independent of preference parameters, $\theta$ and $\beta$, but interest rates are not available for all countries in our sample.

For the second stage estimation of $\sigma$, we extract the long-run trend for each time series $x^i_t$ described above, for each country $i$, by estimating $\beta^i_x$ via ordinary least squares:

$$\log x^i_t = a^i_x + \beta^i_x t + n^i_{x,t}. \quad (16)$$

We then have a triplet of these coefficients, $(\beta^i_s, \beta^i_p, \beta^i_\zeta)$, for each country. This cross-sectional data will be used to estimate $\sigma$ in Equation (15).

data sets. We will present estimates using each labor’s share series.

9The values for $\beta$ and $\delta$ are chosen to be consistent with KN, who assign these values when they estimate their model with markups. Our baseline estimates of $\sigma$ are robust to different values of $\theta$, $\beta$, and $\delta$. Tables with different parameter values are available upon request.

10Dropped country-year observations typically make up less than 2% of country-year observations, as reported in Table 5.

11Results are robust to using lending rates and CPI growth.

12This measure of the long-run trends follows KN.

13We refer to these estimates as $\hat{\beta}^i_x$ rather than $\tilde{\beta}^i_x$ to save notation.
4.2 Sample Selection

Sample selection poses a tradeoff between time series length in the first stage and cross-sectional sample size in the second stage. The threshold for a country’s inclusion in the second stage is the length of their investment price and labor’s share time series, $T_{\text{min}}$. On the one hand, we want enough years of data to calculate long-run trends in the first stage. On the other hand, we want as many countries in the second stage regression as possible, so we do not want to set $T_{\text{min}}$ too high. Our preferred threshold drops countries with fewer than ten years of labor share and investment price data, which leaves us with 56−86 countries for estimating Equation (15), depending on which labor’s share and investment price series we use. We will also discuss KN’s subsample, which uses $T_{\text{min}} = 15$ in the first stage and leaves 36−58 countries for the second stage.\footnote{We prefer $T_{\text{min}} = 10$ because it balances the tradeoff between first and second stage sample sizes. For the PWT investment price series, increasing from $T_{\text{min}} = 10$ to $T_{\text{min}} = 15$ increases the minimal first stage sample size by 50% while reducing from $T_{\text{min}} = 15$ to $T_{\text{min}} = 10$ increases the second stage sample size from 58 to 86 (nearly 50%).}

5 Estimation Results

In order to derive an empirical regression model, we treat $A^i_t$ as an unobserved variable with world-wide trend, $\gamma$, and country specific trend, $\varepsilon^i$:

$$\log A^i_t = \frac{1}{1 - \sigma} \left( \gamma t + \varepsilon^i t \right).$$  \hspace{1cm} (17)

The reduced form regression equation corresponding to Equation (15) is given by:

$$\frac{\bar{s}^i}{1 - \bar{s}^i} \beta^i_s = \gamma + (\sigma - 1) \left( \beta^i_p + \beta^i_\zeta \right) + \varepsilon^i;$$  \hspace{1cm} (18)

We initially assume that the relative trends in capital-augmenting productivity ($\varepsilon^i$) are uncorrelated with average investment price growth and later argue that the correlation between $\varepsilon^i$ and $\beta^i_p$ is most likely negative, which biases our estimates upwards and supports our conclusion that capital-deepening cannot explain the global decline in labor’s share. Since the cross-sectional sample size remains relatively small across all thresholds, we estimate Equation (18) via robust regression in order to minimize the effects of outliers, although we present estimates from other regression models for comparison.
Table 2 report our estimates for each combination of labor’s share and investment price series. The point estimates of $\sigma$ are typically slightly below one, but the sample using corporate labor’s share and WDI investment price data is somewhat larger and pulls the overall average across all series up to $\hat{\sigma} = 1.018$. All estimates are statistically indistinguishable from one, with the exception of the combinations of corporate labor’s share and WDI investment prices.

Following KN, we have estimated the reduced form model using robust regression because our cross-sectional sample sizes are small (at most 86 countries survive the exclusion restriction of $T_{min} = 10$). The robust regression procedure repeatedly fits weighted OLS lines to the data and re-weights observations by their distance from the line. This tends to reduce standard errors, but can also change the sign of $\hat{\sigma} - 1$, as seen in Table (3) where the OLS estimates of $\hat{\sigma}$ are substantially lower than the robust-regression and average $\hat{\sigma} = 0.979$. As an alternative method to reduce influence of outliers, we also report conditional median quantile regression estimates. These point estimates for $\sigma$ fall in between the OLS and robust regression estimates, with an average of $\hat{\sigma} = 0.982$.

Finally, we report estimates using a measure of the real interest rate to proxy for the inter-temporal marginal rate of substitution in Equation (13). The strength of this approach is that it is agnostic about the utility function and rate of time preference. The drawback is that it requires measuring the real interest rate, which is only available for a subset of countries (sample sizes are reduced to 44-71 countries, depending on the time series used). Table (4) compares our baseline estimates to this alternative specification: using real interest rates further reduces the average point estimate to $\hat{\sigma} = 0.998$. We find it reassuring that the estimates are statistically indistinguishable from our baseline in most cases. Furthermore, the interest-rate based estimates of $\sigma$ imply an even smaller role for capital deepening in explaining the global decline in labor’s share.

In summary, an aggregate Cobb-Douglas production function is consistent with cross-country estimates of the aggregate elasticity of substitution. To the extent that point estimates of this elasticity are greater than one, they are not large enough for declining investment prices to have had a large effect on factor income shares. In the next section we use simulations from the theoretical model to understand the differences between our estimates and those of KN, who omit $\beta^i_\zeta$.
5.1 Bias From Omitting $\zeta$

We now provide simulation evidence that omitting $\zeta_i$ is likely to bias estimates of $\sigma$ upwards when it is slightly above one. As previously discussed, the misspecified model may be preferred if the bias is expected to be small since the exact model requires additional functional form and parameter assumptions. We are therefore interested in knowing how large this bias may be in theory.

We think of country $i$’s path beginning from arbitrary values of $K_0$ and $P_0$ as the economy converges to a new steady-state in response to a fall in the price of investment goods. The model is characterized by the following equations:

\begin{equation}
Y_t^i = A\left[\alpha_k(K_t^i)^{\frac{\sigma-1}{\sigma}} + 1 - \alpha_k\right]^{\frac{\sigma}{\sigma-1}}
\end{equation}

\begin{equation}
\frac{u_c(C_{t+1}^i, 1)}{\beta u_c(C_{t+1}^i, 1)} P_t = R_{t+1} + (1 - \delta) P_{t+1}
\end{equation}

\begin{equation}
\log P_t^i = \log P_{t-1}^i + \log \xi_t^i
\end{equation}

\begin{equation}
s_t^i = (1 - \alpha_k)(Y_t^i)^{\frac{1-\sigma}{\sigma}}
\end{equation}

\begin{equation}
1 - s_t^i = \alpha_k\left(\frac{Y_t^i}{K_t^i}\right)^{\frac{1}{\sigma}}
\end{equation}

\begin{equation}
C_t^i + P_t^i X_t^i = Y_t^i
\end{equation}

\begin{equation}
\log \xi_t^i = \rho \log \xi_{t-1}^i + (1 - \rho)\Delta \xi.
\end{equation}

This model is nested in the one described in Section 2. The first five equations are repeated from Section 2 (with the simplifying assumption of inelastic labor supply), while Equation (24) imposes that the country’s goods market clears and Equation (25) specifies the transition path of investment prices, which slowly change by 100\(\Delta \xi\)%.

All parameter values are listed in Table 6. The most important parameter is $\sigma$, which we assume to be slightly above one at $\sigma = 1.01$. We then check whether estimates $\hat{\sigma}$ from two specifications of Equation (15) (with $\zeta$ included and omitted) can accurately recover $\sigma$. We do this by varying our first data point, $t_0 \in \{3,...,10\}$, and then estimate $\sigma$ using twenty years of the transition path.

We begin by plotting time series of the log investment price, log capital’s share, and log $\zeta$ in order to provide intuition for how investment prices and $\zeta$ are predicted to covary along the transition path. These series are found in Figure 2. The investment price falls substantially early on and is then quite flat, while capital’s share rises more gradually.

\footnote{We assume that labor is inelastically supplied for simplicity.}
Clearly, if we were to consider the relative trends in only capital’s share and investment prices and our data began after the first few years, then we would compare an extremely slow decline in investment prices (nearly flat) to a positive trend in capital’s share. This would suggest a large value of \( \sigma \) rather than the value of 1.01 used to generate the series. If we included the trend in \( \zeta \), however, we would realize why capital’s share continues to rise after investment prices flatten out – it is mirrored by a gradual fall in \( \zeta \).

We quantify this intuition by calculating the elasticities directly under the fully specified model (assuming we know the utility function and parameter values) and then under the misspecified model with \( \zeta \) omitted. We calculate \( \sigma \) using the same procedure from the model: we first estimates the growth rates of each variable from \( \log x_t = \beta_0 + \beta x_t + \eta_{xt} \) and then use these growth rates, \( \beta_x \), to calculate \( \sigma \). We define the elasticities calculated from the exact and omitted models as:

\[
\hat{\sigma}_e = 1 - \beta_1 - s / \left[ \beta P + \beta \zeta \right]
\]

\[
\hat{\sigma}_o = 1 - \frac{\beta_1 - s}{\beta P}.
\]

The bias is plotted in Figure 3, measured as \( 100 \left( \frac{\hat{\sigma}_e - \sigma}{\sigma} \right) \). As expected, \( \hat{\sigma}_e \) has essentially zero upward bias regardless of the first period of observation (the flat line in Figure 3), while the misspecified model can generate substantial bias in \( \hat{\sigma}_o \) (the bias grows without bound if the first observation is later in the transition).\(^{16}\)

### 5.2 Why Did Karabarbounis and Neiman Estimate Large \( \sigma \)?

Our estimate of \( \sigma \approx 1 \) may be surprising, since we use the same data and structural model as KN, who consistently estimate \( \sigma \) well above one. As previously mentioned, they omit \( \beta \zeta \) in all of their specifications. In the data, we find substantial cross-sectional volatility in \( \beta \zeta \) and a positive correlation with investment price trends. The expected bias is therefore upward and given by:

\[
\hat{\sigma} - \sigma = (\sigma - 1) \text{corr}(\beta_P, \beta \zeta) \frac{\text{std}(\beta \zeta)}{\text{std}(\beta_P)}
\]

Three terms matter for the sign and size of bias from omitting \( \beta \zeta \): the true value of \( \sigma \), the cross-sectional correlation between \( \beta \zeta \) and \( \beta_P \), and the relative volatility of \( \beta \zeta \).

\(^{16}\)More generally, if \( \beta_P \) and \( \beta \zeta \) are positively correlated, then omitting \( \beta \zeta \) will bias the estimate away from one: if \( \sigma < 1 \), then omitting \( \beta \zeta \) will bias \( \hat{\sigma}_o \) downward.
Figure 4 plots $\beta_i^P$ against $\beta_i^\zeta$ for the sample of countries using the exclusion threshold of KN, $T_{min} = 15$, and the PWT investment price series.\textsuperscript{17} The standard deviation of $\beta_i^\zeta$ is 0.021, where as the standard deviation of $\beta_i^P$ is only 0.013 and the correlation between $\beta_i^\zeta$ and $\beta_i^P$ is 0.18. We therefore expect the bias to be upward when $\sigma > 1$, and larger the farther above one is $\sigma$.

Table 7 compares the estimates from KN’s baseline specification, which omits $\beta_i^\zeta$, to the same specification when $\beta_i^\zeta$ is included.\textsuperscript{18} We use their exclusion restriction of $T_{min} = 15$ and therefore recover their estimates: the average is substantially above one at $\hat{\sigma} = 1.288$, which implies that investment price trends account for over half of the decline in global labor’s share. However, we find evidence that this estimate is biased upward due to the omission of $\beta_i^\zeta$: using the same subsample but including $\beta_i^\zeta$ lowers the average point estimate to $\hat{\sigma} = 1.134$.

Omitting $\zeta$ has a large effect on $\hat{\sigma}$, but sample selection (i.e. varying $T_{min}$) is also important. We therefore find it useful to consider a broad range of values of $T_{min}$ for each specification.\textsuperscript{19} Table 8 reports estimates from KN’s specification, but with the larger cross-sectional sample when $T_{min} = 10$. The point estimates fall substantially and are indistinguishable from one for six of the eight data sets. To fully evaluate the importance of $T_{min}$ in the misspecified model, Figure 5 plots $\hat{\sigma}$ and the 90% confidence intervals for $\sigma$ as we vary $T_{min}$ between 3 and 20.\textsuperscript{20} The point estimates vary substantially, but are below or statistically indistinguishable from one for more than two-thirds of subsamples. Furthermore, estimating Equation (15) including $\beta_i^\zeta$ generates point estimates that are much more robust to $T_{min}$, as seen in Figure (6). These estimates are almost always closer to one than when $\beta_i^\zeta$ is omitted and are estimated with greater precision.

Given the theoretical potential for upward bias from omitting $\beta_i^\zeta$ and the inherent noisiness of real-world data, we are not surprised that different subsamples lead to drastically different estimates of $\sigma$ from the misspecified model. Furthermore, the fact that many subsamples give estimates of $\sigma$ near (or below) one in the misspecified model and that including $\beta_i^\zeta$ always gives smaller and more precise estimates of $\sigma$ suggests that

\textsuperscript{17}The correlation is positive for all series, as seen in the fifth column of Table (7).
\textsuperscript{18}When we use the real interest rate to replace the marginal rate of substitution we get an average of the point estimates of $\hat{\sigma} = 0.98$. We prefer to compare the KN estimates with the baseline $\zeta$ to retain identical samples.
\textsuperscript{19}We will report estimates using robust regression since some exclusion restrictions will lead to even smaller cross-sectional samples. For $T_{min} = 15$ we find that the estimation procedure has a large effect, similar to the case of $T_{min} = 10$. Table 9 shows that standard OLS estimates are much lower and typically indistinguishable from one and, as before, quantile regression estimates are in between.
\textsuperscript{20}This figure uses the PWT/Hybrid series, but similar plots are available for each combination of labor’s share and investment price series.
$\sigma \approx 1$ is an accurate approximation of the aggregate elasticity of substitution between capital and labor.

### 5.3 Capital Specific Technological Change

Thus far, we have assumed that trends in investment prices are independent of trends in capital-augmenting technological progress. We now argue that our conclusion that capital-deepening cannot explain the global decline in labor’s share is robust to allowing correlation between these two trends. Suppose that $x^i \equiv \beta^i_p + \beta^i_{\zeta}$ is correlated with $\varepsilon^i$. The bias in $\hat{\sigma}$ will depend on the true value of $\sigma$ and the sign of this correlation according to:

$$
\hat{\sigma} - \sigma = (1 - \sigma) \text{corr}(x^i, \varepsilon^i) \frac{\text{std}(x^i)}{\text{std}(\varepsilon^i)}.
$$

The most important observation from this equation is that, if $\sigma$ is near one (as our estimates suggest), then we would expect relatively small bias in either direction.\(^{21}\) If the true value of $\sigma$ is above one and if countries with faster declines in investment prices (which implies smaller $x^i$ since $\beta^i_{\zeta}$ is positively correlated with $\beta^i_p$) tend to have faster growth in $A$ (higher $\varepsilon$) then our estimate of $\sigma$ will have upward bias.

This type of bias is also a concern for KN, which they address by calculating the correlation between investment price and total factor productivity trends. They find a negative correlation in these trends across countries, but argue that the resulting bias is small in practice (at most 0.05). We therefore conclude that correlation between $\varepsilon^i$ and $x^i$ is likely to bias $\hat{\sigma}$ upward by a small amount, but that this reinforces our conclusion that capital deepening cannot account for the global decline in labor’s share.

### 6 Conclusion

The aggregate elasticity of substitution between capital and labor ($\sigma$) is an important parameter for understanding trends in factor income shares. The global decline in labor’s share has drawn substantial attention to this parameter - if it is sufficiently above one then the observed decline in investment prices provides a simple explanation for the decline in labor’s share. While aggregate time-series and micro-establishment estimates find $\sigma \leq 1$, previous cross-country estimates of $\sigma$ were much larger than one and implied that more than half of the fall in labor’s share could be explained by capital deepening.

---

\(^{21}\)Clearly, if $\sigma = 1$ then there would be no bias.
in response declining investment prices.

We have shown that previous cross-country estimates of $\sigma$ were subject to omitted variable bias and fragile to sample selection. We provided evidence that the cross-country estimate of $\sigma$ is economically consistent with the conclusions from the large literature on time-series and micro estimates once we proxy for rental rates consistently with theory and estimate $\sigma$ from a larger cross-section of countries. Cross-country data does not support a large role for capital deepening in explaining the global decline in labor’s share.
References


7 Tables

7.1 Comparison of Predicted Global Labor’s Share

Predictions of labor’s share are generated at the country level using observed $\beta_p^i$ and $\beta_\xi^i$, then aggregated by a GDP-weighted average across countries. Global labor’s share and investment price are constructed as GDP-weighted averages across countries. GDP is measured in U.S. dollars at market exchange rates. Change in labor’s share is measured as the difference in the levels of labor’s share in 2010 and 1976.

Table 1: Comparison of Predicted Global Labor’s Share (1976-2010)

<table>
<thead>
<tr>
<th>$\Delta LS$</th>
<th>$\Delta LS_{predicted}$</th>
<th>$\Delta LS_{data}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.068</td>
<td>-0.003</td>
<td>-0.023</td>
</tr>
<tr>
<td>-0.036</td>
<td></td>
<td>-0.036</td>
</tr>
<tr>
<td>0.044</td>
<td>0.337</td>
<td>0.527</td>
</tr>
</tbody>
</table>

σ Estimate: 1.018  1.134  1.288
7.2 Estimates of $\sigma$

All regressions use the left-hand side of Equation (18) as dependent variable. The independent variable is either the right-hand side of Equation (18) or, in the case of the misspecified models, the trend in investment prices alone.

Table 2: Baseline Estimates

<table>
<thead>
<tr>
<th>Labor Share</th>
<th>Investment Price</th>
<th>$\hat{\sigma}$</th>
<th>90% CI</th>
<th>Obs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hybrid</td>
<td>PWT</td>
<td>0.996</td>
<td>[0.957,1.035]</td>
<td>86</td>
</tr>
<tr>
<td>Hybrid</td>
<td>WDI</td>
<td>1.012</td>
<td>[0.978,1.045]</td>
<td>85</td>
</tr>
<tr>
<td>Corporate</td>
<td>PWT</td>
<td>0.971</td>
<td>[0.897,1.044]</td>
<td>56</td>
</tr>
<tr>
<td>Corporate</td>
<td>WDI</td>
<td>1.075</td>
<td>[1.022,1.127]</td>
<td>57</td>
</tr>
</tbody>
</table>

Table 2: Baseline Estimates (continued)

<table>
<thead>
<tr>
<th></th>
<th>PWT</th>
<th>WDI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hybrid, PWT</td>
<td>0.992 [0.959,1.026]</td>
<td>0.996 [0.971,1.020]</td>
</tr>
<tr>
<td>Hybrid, WDI</td>
<td>0.971 [0.920,1.023]</td>
<td>1.131 [1.087,1.174]</td>
</tr>
</tbody>
</table>

Table 3: Comparison of Baseline using Robust, OLS, and Quantile Regressions

<table>
<thead>
<tr>
<th></th>
<th>Robust $\hat{\sigma}$</th>
<th>90% C.I</th>
<th>OLS $\hat{\sigma}$</th>
<th>90% C.I</th>
<th>Quantile $\hat{\sigma}$</th>
<th>90% C.I</th>
<th>Obs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hybrid, PWT</td>
<td>0.996 [0.957,1.035]</td>
<td>0.941 [0.892,0.990]</td>
<td>0.968 [0.940,0.997]</td>
<td>86</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hybrid, WDI</td>
<td>1.012 [0.978,1.045]</td>
<td>0.988 [0.945,1.031]</td>
<td>0.992 [0.966,1.019]</td>
<td>85</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corporate, PWT</td>
<td>0.971 [0.897,1.044]</td>
<td>0.962 [0.880,1.044]</td>
<td>0.930 [0.878,0.983]</td>
<td>56</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corporate, WDI</td>
<td>1.075 [1.022,1.127]</td>
<td>1.013 [0.952,1.075]</td>
<td>1.011 [0.958,1.064]</td>
<td>57</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Comparison of Baseline using Robust, OLS, and Quantile Regressions (continued)

<table>
<thead>
<tr>
<th></th>
<th>Robust $\hat{\sigma}$</th>
<th>90% C.I</th>
<th>OLS $\hat{\sigma}$</th>
<th>90% C.I</th>
<th>Quantile $\hat{\sigma}$</th>
<th>90% C.I</th>
<th>Obs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hybrid, PWT</td>
<td>0.996 [0.959,1.026]</td>
<td>0.941 [0.891,0.991]</td>
<td>0.970 [0.945,0.994]</td>
<td>79</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hybrid, WDI</td>
<td>0.996 [0.971,1.020]</td>
<td>0.986 [0.951,1.021]</td>
<td>0.983 [0.963,1.003]</td>
<td>77</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corporate, PWT</td>
<td>0.971 [0.920,1.023]</td>
<td>0.980 [0.911,1.049]</td>
<td>0.974 [0.913,1.035]</td>
<td>55</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corporate, WDI</td>
<td>1.131 [1.087,1.174]</td>
<td>1.018 [0.961,1.074]</td>
<td>1.030 [0.993,1.068]</td>
<td>55</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Average: 1.018 0.979 0.982
Table 4: Comparison of Baseline Estimates vs. Using Real Interest Rate

<table>
<thead>
<tr>
<th>Sample</th>
<th>Baseline Real Interest Rate</th>
<th>Real Interest Rate†</th>
<th>90% Conf. Interval</th>
<th>Obs.</th>
<th>90% Conf. Interval</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hybrid, PWT</td>
<td>0.996</td>
<td>0.993</td>
<td>0.957, 1.035</td>
<td>86</td>
<td>0.979, 1.007</td>
<td>71</td>
</tr>
<tr>
<td>Hybrid, WDI</td>
<td>1.012</td>
<td>1.017</td>
<td>0.978, 1.045</td>
<td>85</td>
<td>1.003, 1.030</td>
<td>69</td>
</tr>
<tr>
<td>Corporate, PWT</td>
<td>0.971</td>
<td>0.941</td>
<td>0.897, 1.044</td>
<td>56</td>
<td>0.905, 0.977</td>
<td>46</td>
</tr>
<tr>
<td>Corporate, WDI</td>
<td>1.075</td>
<td>1.008</td>
<td>1.022, 1.127</td>
<td>57</td>
<td>0.974, 1.042</td>
<td>46</td>
</tr>
<tr>
<td>OECD and UN Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hybrid, PWT</td>
<td>0.992</td>
<td>0.998</td>
<td>0.959, 1.026</td>
<td>79</td>
<td>0.986, 1.009</td>
<td>65</td>
</tr>
<tr>
<td>Hybrid, WDI</td>
<td>0.996</td>
<td>1.018</td>
<td>0.971, 1.020</td>
<td>77</td>
<td>1.006, 1.031</td>
<td>62</td>
</tr>
<tr>
<td>Corporate, PWT</td>
<td>0.971</td>
<td>0.976</td>
<td>0.920, 1.023</td>
<td>55</td>
<td>0.952, 1.000</td>
<td>46</td>
</tr>
<tr>
<td>Corporate, WDI</td>
<td>1.131</td>
<td>1.1036</td>
<td>1.087, 1.174</td>
<td>55</td>
<td>1.017, 1.055</td>
<td>44</td>
</tr>
<tr>
<td>Average</td>
<td>1.018</td>
<td>0.998</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

† The real interest rate is measured as the interest rate on deposits less the GDP deflator.

Table 5: Country-Year Observations

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Hybrid, PWT</td>
<td>1575</td>
<td>1548</td>
<td>27 (1.7%)</td>
</tr>
<tr>
<td>Hybrid, WDI</td>
<td>1517</td>
<td>1484</td>
<td>33 (2.2%)</td>
</tr>
<tr>
<td>Corporate, PWT</td>
<td>961</td>
<td>952</td>
<td>9 (0.9%)</td>
</tr>
<tr>
<td>Corporate, WDI</td>
<td>952</td>
<td>941</td>
<td>11 (1.2%)</td>
</tr>
<tr>
<td>OECD and UN Data</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hybrid, PWT</td>
<td>1350</td>
<td>1330</td>
<td>20 (1.5%)</td>
</tr>
<tr>
<td>Hybrid, WDI</td>
<td>1306</td>
<td>1281</td>
<td>25 (1.9%)</td>
</tr>
<tr>
<td>Corporate, PWT</td>
<td>918</td>
<td>909</td>
<td>9 (1.0%)</td>
</tr>
<tr>
<td>Corporate, WDI</td>
<td>906</td>
<td>895</td>
<td>11 (1.2%)</td>
</tr>
</tbody>
</table>
7.3 Parameter Values for Simulation

These are the values used to generate Figures 2 and 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>1.01</td>
<td>Average Estimate</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.91</td>
<td>K&amp;N</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.10</td>
<td>K&amp;N</td>
</tr>
<tr>
<td>$\alpha_k$</td>
<td>0.35</td>
<td>Capital Share 0.34</td>
</tr>
<tr>
<td>$A$</td>
<td>0.81</td>
<td>Normalize $Y = 1$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>1.0</td>
<td>Log utility</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.5</td>
<td>Speed of price fall</td>
</tr>
<tr>
<td>$\Delta_\infty$</td>
<td>$-10%$</td>
<td>Eventual price fall</td>
</tr>
</tbody>
</table>
### 7.4 Comparison With Karabarbounis & Neiman

Table 7: Comparison with KN Estimates, $T_{min} = 15$

<table>
<thead>
<tr>
<th>Omitted $\beta^*_P$</th>
<th>$\hat{\sigma}$</th>
<th>90% Conf. Interval</th>
<th>$\hat{\sigma}$</th>
<th>90% Conf. Interval</th>
<th>$\rho(\beta_P, \beta^*_\zeta)$</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hybrid, PWT</td>
<td>1.246†</td>
<td>[1.113,1.380]</td>
<td>1.102</td>
<td>[1.032,1.172]</td>
<td>0.179</td>
<td>58</td>
</tr>
<tr>
<td>Hybrid, WDI</td>
<td>1.291†</td>
<td>[1.177,1.404]</td>
<td>1.155</td>
<td>[1.088,1.223]</td>
<td>0.131</td>
<td>54</td>
</tr>
<tr>
<td>Corporate, PWT</td>
<td>1.322‡</td>
<td>[1.168,1.475]</td>
<td>1.150</td>
<td>[1.069,1.230]</td>
<td>0.220</td>
<td>40</td>
</tr>
<tr>
<td>Corporate, WDI</td>
<td>1.351‡</td>
<td>[1.234,1.468]</td>
<td>1.183</td>
<td>[1.112,1.255]</td>
<td>0.261</td>
<td>38</td>
</tr>
<tr>
<td>OECD and UN Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hybrid, PWT</td>
<td>1.199†</td>
<td>[1.062,1.336]</td>
<td>1.077</td>
<td>[1.004,1.149]</td>
<td>0.194</td>
<td>50</td>
</tr>
<tr>
<td>Hybrid, WDI</td>
<td>1.305‡</td>
<td>[1.198,1.413]</td>
<td>1.160</td>
<td>[1.098,1.222]</td>
<td>0.239</td>
<td>47</td>
</tr>
<tr>
<td>Corporate, PWT</td>
<td>1.254‡</td>
<td>[1.113,1.395]</td>
<td>1.087</td>
<td>[1.010,1.164]</td>
<td>0.222</td>
<td>37</td>
</tr>
<tr>
<td>Corporate, WDI</td>
<td>1.335‡</td>
<td>[1.236,1.435]</td>
<td>1.161</td>
<td>[1.101,1.220]</td>
<td>0.269</td>
<td>36</td>
</tr>
<tr>
<td>Average</td>
<td>1.288</td>
<td></td>
<td>1.134</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

† $\hat{\sigma}$ estimates match Karabarbounis and Neiman’s baseline estimates.
‡ Karabarbounis and Neiman do not report $\hat{\sigma}$ estimates for these series.
Table 8: Comparison of KN Estimates over Exclusion Threshold, $T_{\min}$

<table>
<thead>
<tr>
<th></th>
<th>$T_{\min} = 15$</th>
<th>$T_{\min} = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\sigma}$ $90%$ Conf. Interval</td>
<td>Obs. $\hat{\sigma}$ $90%$ Conf. Interval</td>
</tr>
<tr>
<td>Hybrid, PWT</td>
<td>1.246† $[1.113,1.380]$</td>
<td>58 1.026 $[0.945,1.106]$</td>
</tr>
<tr>
<td>Hybrid, WDI</td>
<td>1.291† $[1.177,1.404]$</td>
<td>54 0.990 $[0.913,1.067]$</td>
</tr>
<tr>
<td>Corporate, PWT</td>
<td>1.322‡ $[1.168,1.475]$</td>
<td>40 1.143 $[0.961,1.325]$</td>
</tr>
<tr>
<td>Corporate, WDI</td>
<td>1.351‡ $[1.234,1.468]$</td>
<td>38 1.283 $[1.170,1.395]$</td>
</tr>
<tr>
<td>OECD and UN Data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hybrid, PWT</td>
<td>1.199† $[1.062,1.336]$</td>
<td>50 1.014 $[0.940,1.087]$</td>
</tr>
<tr>
<td>Hybrid, WDI</td>
<td>1.305‡ $[1.198,1.413]$</td>
<td>47 0.966 $[0.897,1.034]$</td>
</tr>
<tr>
<td>Corporate, PWT</td>
<td>1.254‡ $[1.113,1.395]$</td>
<td>37 1.082 $[0.930,1.233]$</td>
</tr>
<tr>
<td>Average</td>
<td>1.288</td>
<td>1.092</td>
</tr>
</tbody>
</table>

† $\hat{\sigma}$ estimates match Karabarbounis and Neiman’s baseline estimates.
‡ Karabarbounis and Neiman do not report $\hat{\sigma}$ estimates for these series.

Table 9: Comparison of KN Estimates across Estimators, $T_{\min} = 15$

<table>
<thead>
<tr>
<th></th>
<th>Omitted $\beta \zeta$</th>
<th>Robust</th>
<th>OLS</th>
<th>Quantile$^b$</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\sigma}$ $90%$ C.I.</td>
<td>$\hat{\sigma}$ $90%$ C.I.</td>
<td>$\hat{\sigma}$ $90%$ C.I.</td>
<td>$\hat{\sigma}$ $90%$ C.I.</td>
<td></td>
</tr>
<tr>
<td>Hybrid, PWT</td>
<td>1.246† $[1.113,1.380]$</td>
<td>0.958 $[0.766,1.149]$</td>
<td>0.841,1.196</td>
<td>1.076 $[0.935,1.218]$</td>
<td>58</td>
</tr>
<tr>
<td>Corporate, PWT</td>
<td>1.322‡ $[1.168,1.475]$</td>
<td>1.059 $[0.842,1.277]$</td>
<td>0.842,1.277</td>
<td>1.051 $[0.834,1.267]$</td>
<td>40</td>
</tr>
<tr>
<td>OECD and UN Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hybrid, PWT</td>
<td>1.199† $[1.062,1.336]$</td>
<td>0.958 $[0.766,1.149]$</td>
<td>0.841,1.196</td>
<td>1.084 $[0.932,1.236]$</td>
<td>50</td>
</tr>
<tr>
<td>Hybrid, WDI</td>
<td>1.305‡ $[1.198,1.413]$</td>
<td>1.175 $[0.991,1.358]$</td>
<td>0.991,1.358</td>
<td>1.248 $[1.090,1.407]$</td>
<td>47</td>
</tr>
<tr>
<td>Corporate, PWT</td>
<td>1.254‡ $[1.113,1.395]$</td>
<td>0.992 $[0.772,1.212]$</td>
<td>0.772,1.212</td>
<td>1.053 $[0.870,1.236]$</td>
<td>37</td>
</tr>
<tr>
<td>Average</td>
<td>1.273</td>
<td>1.107</td>
<td>1.194</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a$ Estimates reported are for the conditional median.
† $\hat{\sigma}$ estimates match Karabarbounis and Neiman’s baseline estimates.
‡ Karabarbounis and Neiman do not report $\hat{\sigma}$ estimates for these series.
8 Figures

Figure 1: Comparison of Predicted Global Labor’s Share
Figure 2: Transition Paths of KS, Investment Price, and ζ.
Figure 3: Comparison of Estimate Biases
Figure 4: Correlation Between $\beta^i_\zeta$ and $\beta^i_P$. 
Figure 5: Estimates of $\sigma$ From Misspecified Model by $T_{min}$

Figure 6: Estimates of $\sigma$ From Equation (15) by $T_{min}$
9 Appendix 1: Estimating $\sigma$ With Markups

9.1 Extending the Model

We introduce exogenous, time-varying markups to the growth model by assuming that aggregate’s profits are given by:

$$\mu_i^t Y_i^t - u_i^t N_i^t - R_i^t K_i^t$$  \hfill (30)

Where $\mu_i^t$ is the markup. These profits are then lump-sum rebated to households. The equilibrium is therefore as before, except that factor shares are given by:

$$\mu_i^t LS_i^t = (1 - \alpha_k)(Y_i^t)^{\frac{1-\sigma}{\sigma}}$$  \hfill (31)

$$\mu_i^t KS_i^t = \alpha_k \left( \frac{Y_i^t}{K_i^t} \right)^{\frac{1}{\sigma}}$$  \hfill (32)

$$PS_i^t = 1 - LS_i^t - KS_i^t = 1 - \frac{1}{\mu_i^t}$$  \hfill (33)

Where the final equation defines profit’s share of income. Notice that we are still assuming that labor is inelastically supplied, although this has no effect on our analysis.

9.2 Empirical Model

We first write capital’s share to get a log-linear regression equation:

$$1 - \mu_i^t s_i^t = (\alpha_k)^{\sigma} \left( \frac{A_i^t}{\mu_i^t R_i^t} \right)^{\sigma - 1}$$  \hfill (34)

Taking the log difference from $t$ to $t + 1$, assuming constant $A_i^t$, substituting for $R$, and approximating $\log(1 - \mu_i^{t+1}s_i^{t+1})$ around $\mu_i^{t+1} = mu_i^{t} + s_i^{t+1} = s_i^{t}$ yields the estimating equation:

$$\frac{\mu_i^{t+1}s_i^{t+1}}{1 - \mu_i^{t+1}s_i^{t+1}} \left( \beta_s^i + \beta_{\mu_i^t} \right) = \text{cons} + (\alpha - 1) \left( \beta_{\mu_i^t}^i + \beta_{\mu_k^t}^i + \beta_{\mu_k^t}^i \right) + \nu_i$$  \hfill (35)

If we had time series for markups then we could estimate this model. We construct the average growth in markups in two stages. First we calculate the average level of markups and capital’s share and then we impute the trend in these two variables using observable investment rates.

In order to calculate the averages, we will use the fact that capital’s share can be
written in terms of investment when the economy is in steady state. That is:

\[
\frac{R_i K_i}{Y_i} = \frac{P_i^i X_i^i}{Y_i} \left( \frac{1 + \delta - 1}{\delta} \right) \tag{36}
\]

So by assigning values to \( \beta \) and \( \delta \) and identifying the steady-state as the average over of a variable, we know capital’s share for each country from:

\[
KS^i = \left( \frac{P_i^i X_i^i}{Y_i^i} \right) \left( \frac{1 + \delta - 1}{\delta} \right) \tag{37}
\]

Labor’s share is taken from the original data, so we can compute average markup from:

\[
\bar{\mu}^i = \left( KS^i + LS^i \right)^{-1} \tag{38}
\]

This is still not enough to estimate \( \sigma \) – we need information on the average growth rate or trend in capital’s share. We therefore consider an imputation proposed by Karabarbounis & Neiman, in which the trend in capital’s share is equated to the observed trend in the investment rate. That is, we will impute:

\[
\beta_{KS}^i = \beta_{IR}^i \tag{39}
\]

Where \( IR_i^t = \frac{P_i^t X_i^t}{Y_i^t} \). If we observed only steady states, then the change in investment rates would reveal the change in capital’s share. Since the data is generated from a transition path, this imputation may be misleading, as we show below. For now, we take it as given and estimate the model by deriving the implied average growth in the markup. We use:

\[
\mu_t^i = \frac{1}{LS_t^i + KS_t^i} \tag{40}
\]

We then take the log differences of each side:

\[
\Delta \log \mu_{t+1}^i = -\Delta \log \left( LS_{t+1}^i + KS_{t+1}^i \right)
\]

We then approximate the right hand side around \( LS_t^i = LS_t^i \) and \( KS_t^i = KS_t^i \):

\[
\Delta \log \mu_{t+1}^i \approx - \left( \frac{LS_t^i}{LS_t^i + KS_t^i} \Delta \log LS_{t+1}^i + \frac{KS_t^i}{LS_t^i + KS_t^i} \Delta \log KS_{t+1}^i \right)
\]
And finally we replace levels with their averages and log-differences with trends to get the imputed growth in the markup:

$$
\beta^i_\mu \approx - \left[ \frac{LS^i}{LS^i + KS^i} \beta^i_{LS} + \frac{LS^i}{LS^i + KS^i} \beta^i_{KS} \right]
$$

This allows us to estimate $\sigma$ from Equation (35).

9.3 Estimation Results

We present our estimation results in Table 10 for the four basic data sources and each exclusion restriction. The introduction of markups tends to reduce the estimates of $\sigma$ below one. The average across all estimates is $\hat{\sigma} = 0.983$, though we can never reject $\sigma \geq 1$.

9.4 Imputation Accuracy

We now return to the imputation procedure to highlight potential noise and bias that it may generate. We begin by computing the average markup and profit’s share implied by Equation 38. We plot the distribution for this variable when $\beta = 0.91$ and $\delta = 0.10$ using the hybrid labor’s share series and Penn World Table investment prices. The histogram in Figure 7 shows that the average markup in most countries is actually a mark-down, although there is an enormous amount of variation. The distribution of markups implies large average losses for many countries (up to and beyond 50% of GDP), which can be seen in Figure 8.

The extreme values of markups and losses are due to the capital share imputation in Equation (36). This holds exactly in steady state, but in practice we impose it on average along the transition path. The imputation of capital’s share therefore depends on how the economy’s investment rate transitions relative to the true capital share. This is even more important for the imputation of growth rates in capital’s share and markups.

Equation (36) implies that, if we knew that the data began and ended at steady-states, then we could take log-difference of the investment rate between the ending and initial dates. In practice, however, we observe the transition path and do not know if any given year corresponds to the initial or final steady state. Even still, we could approximate the average growth in investment rates with the average growth in capital’s share if the transition paths of these two variables were monotone. We would then get
the right sign for growth in capital’s share even if we missed on the magnitude (if, for example, the two variables converged at different rates). Unfortunately, the transition path of the investment rate is non-monotone in the Neo-Classical Growth model, which we demonstrate by comparing the paths of capital’s share and the investment rate from a simulation with a constant markup.

We consider the case of a true $\sigma = 0.99$ (which is in the 90% CI of most of our estimates, both with and without markups). The remaining parameters can be found in Table 11, along with the rationale for their values. The transition paths log-$KS$ and log-$IR$ are plotted at different horizons in Figure 9. In this economy, capital’s share is declining monotonically by a small amount while the investment rate is hump-shaped with relatively large swings. This type of non-monotonicity must occur if the actual elasticity of substitution is less than one: we know that the steady-state capital’s share must fall, which can only happen if there is capital deepening, which requires at least some period of heightened investment rates. Therefore, in general, the transition path for the investment rate will look quite different from that of capital’s share. Specifically, the investment-rate series may have an upward trend over some time-periods even when $\sigma < 1$ and capital’s share is in fact declining. We conclude from this exercise that the average growth in capital’s share is not generally equal to that of the investment rate, except when comparing the exact change from one steady state to another. Specifically, this equivalence is unlikely to hold if the data is generated from a transition between steady states. We therefore put much less faith in the estimates of $\sigma$ under markups, although they are economically consistent with the estimates without markups.
Table 10: Model with Markups Estimates

<table>
<thead>
<tr>
<th>Hybrid, PWT</th>
<th>Exact</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\sigma}$</td>
<td>90% Conf. Interval</td>
<td>Obs.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{min} = 10$</td>
<td>1.031</td>
<td>[0.976,1.086]</td>
<td>80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{min} = 15$</td>
<td>0.918</td>
<td>[0.852,0.984]</td>
<td>57</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corporate, PWT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{min} = 10$</td>
<td>0.972</td>
<td>[0.905,1.038]</td>
<td>55</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{min} = 15$</td>
<td>0.909</td>
<td>[0.836,0.982]</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hybrid, WDI</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{min} = 10$</td>
<td>1.056</td>
<td>[1.012,1.100]</td>
<td>81</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{min} = 15$</td>
<td>1.017</td>
<td>[0.935,1.099]</td>
<td>54</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corporate, WDI</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{min} = 10$</td>
<td>0.990</td>
<td>[0.921,1.058]</td>
<td>52</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{min} = 15$</td>
<td>0.969</td>
<td>[0.875,1.063]</td>
<td>38</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.983</td>
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</table>
Table 11: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>0.99</td>
<td>Demonstrate error</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.91</td>
<td>K&amp;N</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.10</td>
<td>K&amp;N</td>
</tr>
<tr>
<td>$\alpha_k$</td>
<td>0.40</td>
<td>Capital Share 0.36</td>
</tr>
<tr>
<td>$A$</td>
<td>0.81</td>
<td>Normalize $Y = 1$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>1.0</td>
<td>Log utility</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1.05</td>
<td>Profit share 5%</td>
</tr>
</tbody>
</table>
Figure 7: Distribution of Average Markups
Figure 8: Distribution of Average Profit Shares
Figure 9: Transitional Dynamics of $KS$ and $IR$