

Aggregate Effects of Minimum Wage Regulation at the Zero Lower Bound *

Andrew Glover
University of Texas - Austin

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Abstract

The Fair Minimum Wage Act of 2007 increased the U.S. nominal minimum wage by 41 percent immediately prior to nominal interest rates hitting the Zero Lower Bound in 2008. I study the interaction of these two events in an extension of the sticky-price New Keynesian model. The minimum wage dampens the contractionary effects of the ZLB by preventing rapid wage deflation, halting the deflationary spiral caused by low aggregate demand. For sufficiently persistent ZLB shocks, the minimum wage generates infinite output gains relative to flexible wages, while GDP losses are reduced by half in a calibrated economy. Increasing the minimum wage at the ZLB is expansionary: accumulated output gains are more than 15 percent in the calibrated economy.

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How do aggregate output and employment respond to increases in the minimum wage? How does the minimum wage interact with shocks to the economy? These are fundamentally macroeconomic questions, since the nominal minimum wage has general equilibrium effects on inflation and income, to which monetary policy responds. They are particularly relevant following the Great Recession, which triggered an unprecedented period of near-zero nominal interest rates, but was also preceded by a historically large increase in the minimum wage, especially in real terms (see Figure (1a)). While the aggregate effects of nominal interest rates hitting the zero lower bound (ZLB) has since been extensively analyzed, the interaction between the ZLB and the nominal minimum wage has not. I study this interaction in a parsimonious extension of the sticky-price New Keynesian model that includes heterogeneous workers and nominal minimum wages.

I add two key features to the standard sticky-price New Keynesian model. First, workers have heterogeneous labor inputs that enter production with different productivities and with a general elasticity of substitution. Second, the minimum wage binds for low-productivity workers, which causes their labor supply to be constrained by demand. The resulting macroeconomic model is familiar, but novel. In equilibrium, output, inflation, nominal interest rates, and the real-minimum wage are governed by four dynamic equations: a standard “IS” curve, an augmented Phillips curve, a Taylor Rule, and law of motion for the real minimum wage. The most important deviation from the textbook model is in the Phillips Curve. The presence of a binding minimum wage reduces the slope with respect to output and variation in the real minimum wage shifts the Phillips curve over time.

Monetary policy’s interaction with the minimum wage is key to understanding the aggregate effects of the minimum wage. When the Taylor Rule dictates that nominal rates respond more than one-for-one to inflation, the model predicts that increasing the minimum wage is contractionary. However, if the central bank is willing to accept more inflation in response to a minimum wage increase then output rises with the minimum wage. Since monetary policy’s response is pivotal, the minimum wage is particularly interesting when

nominal interest rates are unresponsive to inflation, as during a zero lower bound episode.

I therefore consider a demand-driven recession in which the central bank lowers nominal interest rates to zero. The existence of a binding minimum wage dampens the negative output and inflation effects of such a shock by halting the deflationary spiral that typically causes a severe contraction during a ZLB episode. Price deflation during a recession requires even faster nominal wage deflation, which the minimum wage effectively outlaws. When nominal wages remain high, firms cut prices less deeply, even in the face of low demand. For a sufficiently persistent ZLB shock, the gains from having a positive share of workers earn a binding minimum wage are unbounded. Modeling the minimum wages therefore extends the model's range of application considerably, since the standard model becomes pathological before the ZLB shock is fully persistent.¹

Quantitatively, accounting for the minimum wage brings the calibrated model closer to the 2009-2015 US experience. The minimum wage is a powerful guard against the ZLB's deflationary spiral: in the calibrated economy, minimum-wage workers receive only 3% of earnings, yet the contractionary effect of the ZLB is reduced by half and deflation is reduced by six percentage points. This reduction in deflation is apropos: as documented by Ball and Malmender [1] and Hall [2], the Great Recession did not bring rapid deflation, as predicted by both traditional Phillips Curve based forecasts and the standard New Keynesian model. While the model with a minimum wage still generates deflation (4.6% in the baseline calibration), it brings the model closer to data.²

Increasing the minimum wage during a zero lower bound episode achieves

¹For example, in the baseline calibration the ZLB is expected to last for ten quarters. In the model without a minimum wage, the ZLB must be expected to bind for under eleven quarters in order to obtain a unique stable solution.

²As shown by Del Negro, Giannoni, and Schorfheide [3], an estimated DSGE model with financial friction predicts disinflation consistent with data. This is partly because they estimate a higher degree of price stickiness than is typically used and therefore a flatter Phillips Curve. Since the minimum wage flattens the Phillips Curve for a given degree of price stickiness, the present mechanism is complementary to theirs.

further output and inflation gains, since the central bank does not follow the contractionary policy dictated by a Taylor Rule. This effect is similar to a wage-inflationary policy at the ZLB, which is expansionary in the standard model due to the “Paradox of Toil” as introduced by Eggertsson [4] (he considers changes in labor supply, but payroll taxes would have the same effect in the standard model). By increasing price inflation, these shocks reduce the real interest rate at the ZLB, which boosts aggregate demand. As I discuss in Section (3.1), the minimum wage strengthens this mechanism: increasing the minimum wage remains expansionary in economies where the ZLB shock is highly persistent, while the standard model admits equilibria where inflationary policies are contractionary at the ZLB.³

More importantly, increasing the minimum wage is strongly expansionary in economies where borrowing frictions reduce the efficacy of policies that reduce the real interest rate. A growing literature finds that borrowing-constraints undermine both conventional and unconventional monetary policy because many households do not respond to the real interest rate. Kaplan, Moll, and Violante [5] show the effect of borrowing constraints for the operation of monetary policy in normal times and McKay, et al [6] show that forward guidance is less effective when households are borrowing constrained. In Section (3.2), I show that increasing the minimum wage is an especially powerful policy in the presence of such frictions.

The minimum wage is a particular type of downward nominal wage rigidity, but is unique in two ways. First, it directly affects only a small share of the labor force, whereas existing models of downward nominal wage rigidities typically constrain all (or most) workers.⁴ Second, and more importantly, the minimum wage is a policy instrument that can be judiciously adjusted in response to aggregate shocks, whereas the existing literature models wage

³The standard model is indeterminate when the ZLB episode is highly persistent.

⁴Schmitt-Grohe and Uribe [7] use a representative agent model in which the aggregate wage is downwardly rigid. Benigno and Ricci [8] build a model with heterogeneous workers, each of whom faces downward rigidity. Erceg, Henderson, and Levin [9] introduce general nominal wage rigidity using monopolistically competitive wage setting by households who face Calvo adjustment costs across workers. Daly and Hobijn [10] adapt this model so a worker can always increase her wage.

rigidity as exogenous and policy invariant.⁵

The paper proceeds as follows: I first describe the extended model, which I use to show the interaction between monetary policy and minimum wage increases. The fact that increasing the minimum wage is more expansionary when the central bank is willing to accept inflation indicates that it is especially powerful when interest rates are fixed. I then study the effect of the minimum wage during a demand-driven recession when the zero lower bound restricts monetary policy, discuss, and conclude.

1 The Model

I extend the textbook sticky-price New Keynesian to include two key features. First, labor is heterogeneous: workers differ in their productivity and labor inputs enter production with a general (constant) elasticity of substitution. Second, low-productivity workers are subject to labor supply constraints. These constraints will be determined by labor demand in equilibrium, which is a reduced form way to capture labor distortions of the minimum wage.⁶

1.1 Model

The model is based on Gali [12]. A continuum of firms produce differentiated goods. These firms compete monopolistically by choosing prices, subject to Calvo [13] price adjustment frictions, which makes the price decision forward looking. A measure of large families comprise two types of workers, low and high productivity. The labor input of these two worker types is aggregated by firms using a CES production function. There are two sources of uncertainty: stochastic increases in the minimum wage and a preference shock which drives the interest rate on risk free bonds to the zero lower bound.

⁵Schmitt-Grohe and Uribe [11] allow the downward wage constraint to respond to output according to an exogenous function, but not directly to policy.

⁶This assumption is necessary for the model to have both a minimum wage policy and for the central bank to follow a monetary policy rule. If all workers are on their labor supply curve then increasing the minimum wage is always expansionary, but monetary policy must be completely passive for an equilibrium to exist.

1.1.1 Household Problem

Households choose individual workers' consumption and labor to solve

$$\max_{c, n_L, n_H} \mathbb{E} \sum_{t=0}^{\infty} \beta^t e^{\zeta_t} \left[(1 - \sigma)^{-1} \left(\int_0^1 c_{i,t}^{\frac{1}{\gamma}} di \right)^{\gamma(1-\sigma)} - \Psi \frac{\nu}{1 + \nu} (\epsilon_H n_H^{\frac{1+\nu}{\nu}} + \epsilon_L n_L^{\frac{1+\nu}{\nu}}) \right]$$

subject to:

$$\int_0^1 p_{i,t} c_{i,t} di + \frac{b_{t+1}}{1 + i_t} = \epsilon_H W_{H,t} n_{H,t} + \epsilon_L W_{L,t} n_{L,t} + b_t + T_t, \quad (1)$$

$$n_{L,t} \leq \underline{N}_{L,t}. \quad (2)$$

This problem is standard except for the constraint $n_{L,t} \leq \underline{N}_{L,t}$. From the household's perspective, the upper bound on low-productivity labor is an exogenous random variable, but in equilibrium it will be set to labor demand (per worker). The optimality choice of each intermediate good $c_{i,t}$ is given by

$$c_i^d(p_{i,t}; P_t) = \left(\frac{p_{i,t}}{P_t} \right)^{\frac{\gamma}{1-\gamma}} C_t, \quad (3)$$

where the aggregate price index is $P_t = \left(\int_0^1 P_{i,t}^{\frac{1}{1-\gamma}} di \right)^{1-\gamma}$ and total consumption is given by

$$P_t C_t = \epsilon_H W_{H,t} n_{H,t} + \epsilon_L W_{L,t} n_{L,t} + b_t + T_t - \frac{b_{t+1}}{1 + i_t}.$$

The inter temporal Euler Equation can then be written in terms of total consumption as

$$C_t^{-\sigma} = \beta \mathbb{E}_t \left[\frac{e^{\zeta_{t+1}}}{e^{\zeta_t}} C_{t+1}^{-\sigma} (1 + i_t) \frac{P_t}{P_{t+1}} \right]. \quad (4)$$

The expectation in this Euler Equation includes ζ_{t+1} , which will drive a demand shock and reduce the natural interest rate below zero. The household has two labor supply conditions, one for each worker type, and must respect the upper bound on low-productivity labor. Per-worker labor supply is there-

fore

$$n_{H,t}^s = \left(\Psi^{-1} \frac{W_{H,t}}{P_t} C_t^{-\sigma} \right)^\nu, \quad (5)$$

$$n_{L,t}^s = \min \left\{ \frac{N_{L,t}}{P_t}, \left(\Psi^{-1} \frac{W_{L,t}}{P_t} C_t^{-\sigma} \right)^\nu \right\}. \quad (6)$$

1.1.2 Firm Profit Maximization

Firms compete monopolistically by choosing the price of their good, which they can reset with a constant probability in each period. The firm's price in a given period t determines the demand for their good, which they must meet by hiring labor. This leads the firm to choose labor inputs in the least costly way possible subject to their goods demand. The profit maximization problem for a price setting firm is:

$$\begin{aligned} \max_{p, N_L, N_H} \mathbb{E}_t \sum_{\ell=0}^{\infty} \lambda^\ell \frac{\beta^\ell e^{\zeta_{t+\ell}} C_{t+\ell}^{-\sigma}}{e^{\zeta_t} C_t^{-\sigma}} & \left(p c_{i,t+\ell}^d(p; P_{t+\ell}) - w_{L,t} N_{L,t} - w_{H,t} N_{H,t} \right) \\ & \text{subject to:} \\ c_{i,t+\ell}^d(p; P_{t+\ell}) & \leq \left(\theta N_{L,t+\ell}^{\frac{\eta-1}{\eta}} + N_{H,t+\ell}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \forall \ell \geq 0 \end{aligned} \quad (7)$$

Where $1 - \lambda$ is the probability that a firm is allowed to change their price in a given period. The firm is owned by households, so the stochastic discount factor is derived from the household's marginal rate of substitution. A firm that is not able to adjust its price in period t minimizes the cost of production to meet demand in that period, taking nominal wages as given. Given a price p in period $t + \ell$, the firm's labor demand conditions are given by

$$\frac{N_{i,H,t+\ell}^d}{N_{i,L,t+\ell}^d} = \left(\frac{W_{L,t+\ell}}{\theta W_{H,t+\ell}} \right)^\eta, \quad (8)$$

$$c_{i,t+\ell}^d(p; P_{t+\ell}) = N_{i,L,t+\ell}^d \left(\left(\frac{N_{H,t+\ell}}{N_{L,t+\ell}} \right)^{\frac{\eta-1}{\eta}} + \theta \right)^{\frac{\eta}{\eta-1}}. \quad (9)$$

Equation (8) implies that the ratio of labor inputs is independent of a firm's price and the demand for high-productivity workers relative to low is positively related to the relative wage. If the relative wage of low-productivity workers rises by 1% (through minimum wage policy, for example) then the relative demand for low-productivity workers falls by η percent. Equation (9) then determines the scale of the firm's labor force based on total goods demand.⁷ The firm's price setting problem simplifies to:

$$\max_p \mathbb{E}_t \sum_{\ell=0}^{\infty} \lambda^\ell \frac{\beta^\ell e^{\zeta_{t+\ell}} C_{t+\ell}^{-\sigma}}{e^{\zeta_t} C_t^{-\sigma}} \left((p - \mathcal{W}_{t+\ell}) c_{i,t+\ell}^d(p; P_{t+\ell}) \right) \quad (10)$$

Where the nominal marginal cost of production, \mathcal{W}_t , is defined by combining Equations (8) and (9):

$$\mathcal{W}_t = \frac{W_{L,t}}{\theta} \left(\left(\frac{W_{L,t+\ell}}{\theta W_{H,t+\ell}} \right)^{\eta-1} + \theta \right)^{-\frac{1}{\eta-1}}. \quad (11)$$

The price chosen at period t that achieves the above maximal profit above is given by:

$$P_t^\# = \gamma \frac{\mathbb{E}_t \left[\sum_{\ell=0}^{\infty} \lambda^\ell Q_{t,t+\ell} \mathcal{W}_{t+\ell} \right]}{\mathbb{E}_t \left[\sum_{\ell=0}^{\infty} \lambda^\ell Q_{t,t+\ell} \right]}, \quad (12)$$

where $Q_{t,t+\ell} = \frac{\beta^\ell e^{\zeta_{t+\ell}} C_{t+\ell}^{-\sigma}}{e^{\zeta_t} C_t^{-\sigma}}$ is the stochastic discount factor from t to $t + \ell$. This pricing equation and the labor demand equations then give profits for firm i in period t as $T_{i,t}$.

1.1.3 Policy and Shocks

Monetary policy is governed by a Taylor Rule on nominal interest rates, which are subject to the zero lower bound. The shocks in this economy do not affect the flexible-price level of output, so I set the inflation target to zero and the

⁷The ratio of labor inputs is independent of a firm's output for any constant-returns production function.

output target to steady state.

$$\log(1 + i_t) = \max \left\{ -\log \left(\beta \mathbb{E}_t \frac{e^{\zeta_{t+1}}}{e^{\zeta_t}} \right) + \psi_y \log \frac{Y_t}{Y_{ss}} + \psi_\pi \log \frac{P_t}{P_{t-1}}, 0 \right\}. \quad (13)$$

The minimum wage is modeled as the prevailing nominal wage for low-productivity workers, $W_{L,t}$ and follows the following stochastic process

$$\log W_{L,t} = \log W_{L,t-1} + \tau \log \left(\frac{P_t}{P_{t-1}} \right) + m_t, \quad (14)$$

with the shock $m_t > 0$.

The shock to preferences follows a process with two regimes. In the first (non-ZLB) regime, $\zeta_{t+1} = \zeta_t$. In the second (ZLB) regime, the natural rate of interest is negative:

$$-\log \left(\beta \mathbb{E}_t \frac{e^{\zeta_{t+1}}}{e^{\zeta_t}} \right) = r^{zlb} < 0. \quad (15)$$

The probability of switching from the first regime to the second is zero while the probability of switching from the second to the first is $1 - \chi$.

1.1.4 Aggregation and Market Clearing

The low-productivity labor demand for firm i is given by:

$$N_{i,L,t}^d = \Omega_t^{-1} c_{i,t}^d(p_{i,t}; P_t) = \Omega_t^{-1} \left(\frac{p_{i,t}}{P_t} \right)^{\frac{\gamma}{1-\gamma}} C_t, \quad (16)$$

where Ω_t is given by

$$\Omega_t = \left(\left(\frac{W_{L,t}}{\theta W_{H,t}} \right)^{\frac{\eta-1}{\eta}} + \theta \right)^{\frac{\eta}{\eta-1}}. \quad (17)$$

Setting $C_t = Y_t$ and integrating over all firms gives aggregate labor demand relative to the nominal unit-cost of production, aggregate consumption, and price dispersion:

$$N_{L,t}^d = \Omega_t^{-1} \Delta_t Y_t. \quad (18)$$

Where $\Delta_t = \int_0^1 \left(\frac{p_{i,t}}{P_t}\right)^{\frac{\gamma}{1-\gamma}} di$ is the wedge generated by cross-sectional price dispersion at t . Labor demand also determines the labor supply constraint on low-productivity workers

$$\underline{N}_{L,t} = \epsilon_L^{-1} N_{L,t}^d, \quad (19)$$

and high-productivity demand follows from

$$N_{H,t}^d = \left(\frac{W_{L,t}}{\theta W_{H,t}}\right)^\eta N_{L,t}. \quad (20)$$

Setting aggregate profits to $T_t = \int_0^1 T_{i,t} di$ and imposing bond market clearing in each period ensures that the budget constraint is consistent with goods market clearing. The laws of motion for aggregate prices and price dispersion are then given by

$$P_t^{\frac{1}{1-\gamma}} = \lambda P_{t-1}^{\frac{1}{1-\gamma}} + (1-\lambda)(P_t^\#)^{\frac{1}{1-\gamma}}, \quad (21)$$

$$\Delta_t = (1-\lambda) \left(\frac{P_t^\#}{P_t}\right)^{\frac{\gamma}{1-\gamma}} + \lambda \left(\frac{P_{t-1}}{P_t}\right)^{\frac{\gamma}{1-\gamma}} \Delta_{t-1}. \quad (22)$$

1.2 Steady-State

In steady-state all real variables are constant, the inflation rate is zero, $\zeta = 0$ is constant and $m_t = 0, \forall t$. Since there is no inflation the price-dispersion wedge is $\Delta = 1$. The list of steady-state equations can be found in the appendix. The system is complicated by Equation (6). If the minimum wage does not bind in steady state then $N_L = \left(\Psi^{-1} \frac{W_L}{P} Y^{-\sigma}\right)^\nu$, i.e. both households are on their labor supply curve and the equilibrium determines real wages (the price level is still indeterminate). If the low-productivity labor supply constraint binds then the low-productivity household is constrained and demand determines her labor in equilibrium, so policy must provide the final restriction for equilibrium. While the nominal minimum wage is the policy instrument away from steady state, the *real* minimum must be chosen to determine a steady-state equilibrium. I will linearize around the steady-state where the labor-supply

constraint just binds, which means that

$$\frac{\epsilon_H}{\epsilon_L} \left(\frac{W_L}{\theta W_H} \right)^\eta \left(\Psi^{-1} \frac{W_H}{P} Y^{-\sigma} \right)^\nu = \left(\Psi^{-1} \frac{W_L}{P} Y^{-\sigma} \right)^\nu. \quad (23)$$

Since the nominal minimum wage process is increasing, I use the first term to linearize around the steady-state. The following analytical results are unchanged if the initial steady-state has a larger binding minimum wage.

1.3 Linearization

Log-linearizing the economy around a steady-state with binding minimum wage yields a “four-equation” model

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma} (i_t - r_t - \mathbb{E}_t \pi_{t+1}), \quad (24)$$

$$i_t = \max\{0, r_t + \psi_y y_t + \psi_\pi \pi_t\}, \quad (25)$$

$$w_{L,t}^r = w_{L,t-1}^r - \pi_t + m_t, \quad (26)$$

$$\pi_t = \left(\frac{\nu(1 - s_L)\kappa}{\eta s_L + \nu} \right) y_t + \left(\frac{\nu s_L \kappa (\eta + \nu)}{(1 + \sigma\nu)(\eta s_L + \nu)} \right) w_{L,t}^r + \beta \mathbb{E}_t \pi_{t+1}, \quad (27)$$

where $r_t = \frac{1-\beta}{\beta} - (\mathbb{E}_t \zeta_{t+1} - \zeta_t)$ is the natural rate of interest at t and $w_{L,t}^r = w_{L,t} - p_t$ is the real minimum wage. The derivation of this system is in the appendix. The first equation is the New Keynesian IS curve, the second is a Taylor Rule representing monetary policy. These two equations are standard. The third equation is the law of motion for the real minimum wage, which follows from the process for the nominal minimum wage: the log-deviation of the nominal minimum wage from steady state is $w_{L,t} = w_{L,t-1} + m_t$ and $p_t = p_{t-1} + \pi_t$. The final equation is a modified Phillips curve, which is the most novel feature and warrants further discussion.

The Phillips curve depends on the composite parameter $\kappa = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda} \frac{1+\sigma\nu}{\nu}$, but also on the elasticity of substitution between labor types (η) and the fraction of output created by low-productivity workers (s_L). These two parameters dictate how this Phillips curve differs from the standard one without minimum wages. The elasticity parameter governs how much a change in $w_{L,t}$ passes

through to $w_{H,t}$ in equilibrium. If the two worker types are perfect substitutes then $\eta = \infty$ and the weight on output in the Phillips curve goes to zero; the model is identical to a representative agent model in which the nominal wage is set by policy. When the two labor types are not perfect substitutes the high-productivity wage does not respond one-for-one to the minimum wage, so the share of output created by each labor input matters. In that case, the larger is s_L the less output feeds back to inflation since fewer workers are on their labor supply curves. Finally, if the real minimum wage is set so that low-productivity workers' intratemporal Euler Equation holds (i.e., $w_{L,t} - p_t = \frac{1+\sigma\nu}{\nu}y_t$) then the Phillips curve collapses to the standard $\pi_t = \kappa y_t + \beta \mathbb{E}_t \pi_{t+1}$.

2 Quantitative Results

Calibrating this economy requires setting two new parameters, η and s_L , since together they determine the weights on output and the real minimum wage in the Phillips Curve. I calibrate the elasticity of substitution using wage and earnings responses to changes in the minimum wage. I calibrate s_L using the share of earnings accruing to workers near the minimum wage.

For the elasticity of substitution, η , I use Equation (8) along with microeconomic estimates of the effect of increasing the minimum wage on earnings for workers earning wages at and above the minimum. Equation (8) relates changes in the the minimum-wage-earnings ratio to the minimum-wage ratio as

$$\Delta \log (W_{L,t}N_{L,t}) - \Delta \log (W_{H,t}N_{H,t}) = (1-\eta) \left(\Delta \log W_{L,t} - \Delta \log W_{H,t} \right). \quad (28)$$

The required statistics to compute η are reported in Neumark, Schweitzer, and Wascher [14]. They estimate the response of wages and earnings to a 1% increase in the minimum wage for workers earning up to 800% of the minimum. Their estimates imply that, in response to a 1% increase in the minimum wage, the term $\log W_{L,t} - \log W_{H,t}$ rises by 0.71% while the relative earnings of

minimum wage workers falls by 0.05%.⁸ Solving for η in $-0.01 = (1 - \eta)0.71$ gives $\eta = 1.02$.⁹

I use data on earnings shares to calculate s_L , under the assumption that the minimum wage just binds for those earning it in steady-state.¹⁰ In a steady-state where the minimum wage just binds, the natural moment for s_L is the share of earnings accruing to minimum wage workers. However, the effect of the minimum wage depends on the earnings share of workers for whom it binds *plus the earnings share of workers who are perfect substitutes for these workers*. The estimates from Neumark, et al [14] imply that workers earning up to 110% of the minimum wage are near perfect substitutes for minimum wage earners (their elasticity of substitution is $\eta = 20$). I therefore use the share of earnings of workers earning up to 110% of the minimum wage, which gives $s_L = 0.03$ according to the data used by Neumark, Schweitzer, and Wascher.

I set the parameters $\sigma = 2, \nu = 1, \beta = 0.99$ and $\kappa = 0.02$ so that my results are comparable to previous quarterly calibrations of the sticky-price model. I begin with traditional values for the Taylor Rule parameters, $\psi_\pi = 1.5$ and $\psi_y = 0.125$. The central bank might deviate from its normal response if inflation is created consciously by minimum wage policy, so I explore the effects as monetary policy becomes less responsive.

2.1 Effect of Minimum Wage Increase

Figure (1) plots the impulse responses of aggregate output (consumption), inflation, and interest rates to a 10% increase in the nominal minimum wage when the central bank follows a Taylor Rule with strong responses to inflation and output ($\psi_\pi = 1.5$ and $\psi_y = 0.125$).¹¹ Inflation increases as firms raise

⁸In order to be consistent with computing s_L , I treat workers earning up to 110% of the minimum wage as low-productivity when computing η .

⁹This is analogous to estimating η using two-stage least squares with the nominal minimum wage as an instrument for $\log W_{L,t} - \log W_{H,t}$.

¹⁰Note that this does not require that the minimum wage bind for these workers in steady state, just that any increase from steady state would cause it to bind.

¹¹This is a large shock relative to typical business cycle fluctuations, but small relative to typical increases in the nominal minimum wage. As previously mentioned, the Fair Minimum Wage Act of 2007 increased the US federal minimum wage from \$5.15 to \$7.25

their prices in face of a higher nominal unit cost of production, which induces the central bank to raise the nominal interest rate. The net effect is a slight increase in the annual real rate, which causes output to fall by 17 basis points. Over time the price level rises by 10% (so that the real unit cost returns to steady-state) and output returns to steady state.

The contraction in output is entirely due to the Taylor Rule responding sharply to inflation, as can be seen in Figure (2) when $\psi_\pi = 0.75$. Increasing the minimum wage now causes more inflation relative to the nominal interest rate so that the real rate falls by eight basis points. This causes output to rise by 35 basis points. The return to steady-state is much more rapid in this case, since the central bank accepts higher inflation. These experiments show that the central bank has ultimate power in shaping the aggregate effect of an increase in the minimum wage.

2.2 Interaction of ZLB and Minimum Wage

The existence of minimum wage interacts with disinflationary shocks, specifically aggregate demand shocks that drive the economy to the zero-lower bound on nominal interest rates. In order to explore this interaction, I now study output and inflation when the economy starts in the ZLB regime. The economy is linearized around the non-ZLB regime steady-state, so that the log-linearized variables all return to zero when the ZLB episode ends.¹² I set the probability of the ZLB persisting to $\chi = 0.9$ so it is expected to last for ten quarters.

The baseline calibration is directly comparable to Eggertsson and Woodford [15] (henceforth, EW), who study the ZLB shock in a representative agent

(about 41%).

¹²This includes the real minimum wage, which means that the nominal minimum wage is adjusted by the total change in the price level once the ZLB episode ends.

with fully flexible wages. The system during the ZLB period is described by

$$y_t = \chi y_{t+1} + \sigma^{-1} r^{zlb} + \sigma^{-1} \chi \pi_{t+1}, \quad (29)$$

$$\pi_t = \left(\frac{\nu \kappa (1 - s_L)}{\eta s_L + \nu} \right) y_t + \left(\frac{\nu s_L \kappa (\eta + \nu)}{(1 + \sigma \nu) (\eta s_L + \nu)} \right) w_{L,t}^r + \beta \chi \pi_{t+1}, \quad (30)$$

$$w_{L,t}^r = w_{L,t-1}^r + (\tau - 1) \pi_t. \quad (31)$$

Writing this system in state-space form, the transition matrix has two eigenvalues outside of the unit circle and one inside: an initial condition on $w_{L,0} - p_0$ is required to determine the solution, which is provided by the policy for the nominal minimum wage. Contrast this to the model of EW, which (for this calibration) has a unique jump solution with rapid deflation of 10.5% per year and a deep recession (output falls by 14.3%). In that model, output and inflation jump to their ZLB values and are constant throughout the episode. The real wage falls by three times output (since $\frac{1+\sigma\nu}{\nu} = 3$), which means that the nominal falls by 45.7% at the onset of the ZLB and then 10.5% per year thereafter. The heterogeneous labor economy without a minimum wage has the same response as the homogeneous labor economy (the solution can also be obtained by fixing $w_{L,t} - p_t = -43.03\%$ in the system above).

Since the economy without a minimum wage exhibits wage deflation, the economy with a just-binding minimum wage will have the constraint $w_{L,t} \geq 0$ bind during the ZLB episode. If the minimum wage just binds for low-productivity workers at the onset of the ZLB episode (as would be the case starting in steady state), then the initial condition is $w_{L,0} - p_0 = -\pi_0$, which generates the path of output and inflation seen in Figure (3). While the economy still experiences a deflationary recession, the depth is greatly reduced relative to the economy without a minimum wage. Inflation falls to -4.6% on an annual basis and output losses are 7.2% on impact, which is large in absolute terms but only half of the decline without a minimum wage. Furthermore, inflation and output rise over time in this economy, which means that accumulated output losses over ten quarters amount to 69.9% rather than 143.4% in the EW model. The accumulated gains from the minimum wage grow with duration, since inflation converges to zero and the output loss ap-

proaches -2.5% .

As the ZLB episode persists in the minimum wage economy, $\pi_t \rightarrow 0$ according to Equation (31), and output converges to:

$$y^{zlb} \equiv \lim_{t \rightarrow \infty} y_t = \frac{\sigma^{-1} r^n}{1 - \chi} \quad (32)$$

This limit is independent of the share of minimum wage workers and finite for $\chi < 1$. This contrasts sharply with the EW model, where output jumps to:

$$y^{ew} = \frac{\sigma^{-1} r^n}{1 - \chi - \frac{\sigma^{-1} \kappa}{1 - \beta \chi}} \quad (33)$$

The output gap in the EW economy asymptotes to $-\infty$ as the persistence of the ZLB episode rises towards the value χ^* defined by $\kappa = (1 - \chi^*)(1 - \beta \chi^*) \sigma (\chi^*)^{-1}$. The minimum wage therefore has a larger dampening effect as $\chi \rightarrow \chi^*$, since output losses are unbounded without it. For the calibrated economy the effect is seen in Figure (4), which plots the paths of output and inflation with ZLB persistence set at $\chi^* = 0.9091$. The initial decline in output is 8.7% and the limit for output is -2.75% , which is infinitely smaller than in the economy without a minimum wage.¹³

To understand the difference between these two economies, consider the firm's price setting problem during the ZLB period. In the standard model, low demand induces firms to lower prices, which is optimal because the nominal wage falls even more than prices. The real wage therefore falls, which further reduces demand. When the minimum wage outlaws nominal wage deflation this cycle is interrupted: firms do not cut prices as deeply because their nominal unit cost does not fall as much.¹⁴

This exercise shows that the ZLB's negative consequences are diminished greatly even if a small share of the population is constrained by the minimum

¹³The plots are made setting $\chi = \chi^*$. In the EW economy the output gap is undefined for this value, but approach $-\infty$ as χ increases towards χ^* .

¹⁴The nominal wage of high-productivity workers falls during the ZLB episode, which causes firms to shift demand towards high-productivity labor. Therefore, the unit nominal cost of production falls in this model, but by less than without a minimum wage.

wage, but the U.S. actually raised the minimum wage just prior to interest rates hitting zero. I now use the model as a laboratory to ask what would have happened if the Fair Minimum Wage Act of 2007 had not been passed at that time.

2.3 Raising the Minimum Wage At The ZLB

Increasing the minimum wage during a ZLB episode is especially effective since the nominal interest rate does not rise in response. I now consider two such policies. In the first experiment I increase the nominal minimum at the onset of the ZLB episode and keep it constant until the ZLB episode ends at which point I return the real minimum wage to steady-state. Therefore, if the ZLB shock ends in period t then $w_{L,t+j} = p_{t+j}$ from then on (this ensures that the economy returns to steady-state once the ZLB episode ends). In the second experiment the minimum wage is indexed to the price level during the ZLB in order to increase the *real* minimum wage by $w_L^r\%$. That is, for any $t > 0$ while the ZLB persists we have that $w_{L,t} - p_t = w_L^r\%$ and zero once the episode ends.

The effect of increasing the nominal minimum wage is plotted in Figure (5), which plots the differences in output and inflation between the economy with a 40.8% increase in the nominal minimum relative to the economy with $w_{L,0} = 0$. This increase is chosen to match the nominal increase legislated by the Fair Minimum Wage Act of 2007, which increased the federal minimum wage from \$5.15 to \$7.25. Increasing the minimum wage by 40.8% at the onset of the ZLB reduces output losses by 1.6%–pts at onset and cuts deflation by a similar amount. The relative output gains tend towards zero as the the ZLB persists, though the output gain remains around 1.5% after ten quarters (the expected duration of the ZLB episode).

If the nominal minimum wage is indexed to the price level then $w_{L,t}^r$ is replaced with the constant w_L^r in Equation (30) and Equation (31) is dropped from the system. There are two values of w^r of particular interest. If $w^r = -43.03\%$ then the the ZLB causes the economy to jump to the EW solutions. If

$w^r = 154.5\%$, then output losses from the ZLB are completely offset (inflation is 2.2%). As w^r varies between these two values there is a monotonic reduction in output losses and deflation, as seen in Figure (6).

3 Discussion

Before concluding, I discuss how the minimum wage relates to other policies that have been studied at the zero lower bound as well as how it affects economies for which the complete markets IS curve is violated.

3.1 Paradox of Toil

Eggertsson [4] introduced the “Paradox of Toil”, which demonstrates that wage-reducing shocks to the labor market can be contractionary at the zero lower bound, even if those shocks are expansionary in normal times (Eggertsson [4] considers an increase in labor supply). In my model, increasing the minimum wage at the ZLB works in an expansionary direction by increasing inflation and reducing the real interest rate. However, the effect is complicated because the real minimum wage responds endogenously to the inflation caused by increasing the minimum wage. The aggregate effect of this feedback was not previously known and has been debated in discussions of the Paradox of Toil. Mulligan [16] argued that increasing the minimum wage as a test of the Paradox of Toil, while Eggertsson [17] claimed that it was not a valid test because the counterfactual is unobservable and because minimum wages have differential effects across the wage distribution. The model from Section (1.1) is built so that the minimum wage directly affects only low-wage earners and provides a laboratory to assess the counterfactual. As shown in Section (2.3), the minimum wage is expansionary at the zero lower bound.

To better understand how the minimum wage interacts with the Paradox of Toil, it is useful to compare inflationary shocks to the Phillips Curve across the two models. Specifically, I consider an inflationary Phillips curve shock in a region where the Paradox of Toil is inactive in the standard model – when

the ZLB shock is highly persistent.¹⁵ This involves adding a term $\tau > 0$ to the right-hand side of Equation (30) for the duration of the ZLB episode, the effects of which are plotted in Figure (7). In the model without a minimum wage (i.e. $s_L = 0$), the jump-solutions for output and inflation fall due to the introduction of τ .¹⁶ In the model with a minimum wage, however, the inflationary shock is expansionary throughout the ZLB episode.

3.2 Effect of Borrowing Constraints

Output is driven by the intertemporal Euler Equation in the model presented thus far, which means that the minimum wage's effect on demand is purely through real interest rates. However, a growing literature argues that much of the real effect of monetary policy arises through labor markets and income effects, because many households are borrowing constrained or otherwise behave counter to the intertemporal Euler Equation.¹⁷ I therefore enrich the model to include this mechanism by introducing a share of households who have high productivity, but consume in a hand-to-mouth fashion. For these households, consumption solves the intra-temporal Euler Equation:

$$(C_t^{HM})^{-\sigma} \frac{W_{H,t}}{P_t} = \Psi (C_t^{HM})^{\frac{1}{\nu}} \left(\frac{W_{H,t}}{P_t} \right)^{-\frac{1}{\nu}}, \quad (34)$$

and thus, using the equilibrium expression for $\frac{W_{H,t}}{P_t}$, the hand-to-mouth household's consumption deviates from steady state according to

$$c_t^{HM} = \left(\frac{1 + \sigma\nu}{\eta s_L + \nu} \right) y_t + \left(\frac{1 + \nu}{1 + \sigma\nu} \right) \left(\frac{\eta s_L}{\eta s_L + \nu} \right) w_{L,t}^r. \quad (35)$$

¹⁵With the minimum wage, the equilibrium is unique, but the equilibrium is indeterminate for highly persistent ZLB shocks in the standard model *without a minimum wage*. My comparative statics are with respect to the constant inflation equilibrium in the standard model. Mertens & Ravn [18] show that the equilibrium is determinate in this region if the ZLB episode is caused by pessimistic expectations instead of a fundamental demand shock. Further, they show that the Paradox of Toil is absent in their model.

¹⁶Note that the jump equilibrium values of output and inflation are positive rather than negative in this region of the parameter space, so the ZLB episode is no longer a recession but instead an expansion. The inflationary shock reduces output and inflation towards zero.

¹⁷See Kaplan, Moll, and Violante [5] and McKay, Nakamura, and Steinsson [6].

The remaining household's consumption still follows from their intertemporal Euler Equation, and total consumption is just the sum of the two. This means that increasing the minimum wage during the ZLB still affects output through demand, but now in two ways. As before, it creates inflation, which decreases the real interest rate, spurring demand by households for whom the Euler Equation is satisfied. But now, it increases wages directly, which raises consumption demand for hand-to-mouth households.

It is straight forward to derive an augmented IS curve in this economy by adding the consumption demand for each household type. Denoting the steady-state share of consumption of hand-to-mouth households by $1 - s_c$, the log-linearized intertemporal Euler Equation for all other households can be written in terms of output and the real minimum wage as

$$y_t = \mathbb{E}_t y_{t+1} - \psi_w \left(\mathbb{E}_t w_{L,t+1}^r - w_{L,t}^r \right) - \psi_R \left(i_t - r_t - \mathbb{E}_t \pi_{t+1} \right), \quad (36)$$

where the constants are given by:

$$\psi_w = s_c \left(\frac{1 + \nu}{1 + \sigma \nu} \right) \left(\frac{\eta s_L}{\eta s_L + \nu} \right) \left(1 - s_c \frac{1 + \nu}{\eta s_L + \nu} \right)^{-1} \quad (37)$$

$$\psi_R = (1 - s_c) \sigma^{-1} \left(1 - s_c \frac{1 + \nu}{\eta s_L + \nu} \right)^{-1}. \quad (38)$$

This IS curve is similar to the “discounted Euler Equation” introduced by McKay, Nakamura, and Steinsson [19], which itself is a reduced form for borrowing constraints as studied by McKay, Nakamura, and Steinsson [6]. When some households are intertemporally constrained, the ZLB episode is directly dampened because the preference shock only reduces demand for a fraction of the population. It also dampens the feedback from deflation to output, since the consumption of hand-to-mouth households does not respond to the real interest rate. These two mechanisms reduce the severity of the ZLB episode in the McKay, Nakamura, and Steinsson model, while also reducing the effect of inflationary shocks to the Phillips Curve since, again, output responds less to changes in the real interest rate. The novelty of the model

outlined here is that the real wage of high-productivity workers determines demand for hand-to-mouth households. On the one hand, this exacerbates the ZLB contraction because wages (and therefore earnings) fall rapidly with output (the $\frac{1+\sigma\nu}{\eta s_L+\nu}y_t$ term in Equation (35)). On the other hand, the real minimum wage rises during the ZLB episode, which raises demand for these households (the second term in Equation (35)).

In order to assess the effect of hand-to-mouth households' demand, I fix the share of consumption for hand-to-mouth households at $s_c = 0.133$, which corresponds to the lower bound for the share of wealth hand-to-mouth households estimated by Kaplan, Violante, and Weidner [20]. I then solve the model outlined above, both with a constant nominal minimum wage and an increase of 40.8%, as in the baseline model. As shown in Figure (8), deflation is sharper and output losses are larger with hand-to-mouth households, which is due to a larger fall in demand for hand-to-mouth households because their wages fall initially. However, as the ZLB episode persists, the real minimum wage rise increases, which increases the earnings and therefore demand for hand-to-mouth households, which causes the paths of output and inflation to be steeper in the model with hand-to-mouth households.¹⁸

While hand-to-mouth households cause larger output losses during the ZLB, they increase the expansionary effect of raising the nominal minimum wage during the ZLB. This can be seen from the solid blue lines in Figure (9), which plots the output and inflation gains from increasing the nominal minimum wage by 40.8% at the onset of the ZLB, both with and without hand-to-mouth households. In the model without hand-to-mouth households, increasing the minimum wage reduces accumulated output losses by 15.3% over ten quarters (the expected duration of the ZLB), which increases to 24.1% in the model with hand-to-mouth households. It also reduces deflation, as seen in the second sub-plot.

Furthermore, raising the minimum wage has a larger effect in the hand-

¹⁸The model with a discounted Euler Equation ala McKay, Nakamura, and Steinsson [19] has dampened output losses and deflation throughout the ZLB episode because the IS curve is missing the real wage term.

to-mouth model than an equivalent inflationary shock to the Phillips Curve. Figure (10) plots output and inflation gains during the ZLB episode with and without hand-to-mouth households when the Phillips curve is shocked by $\tau = 0.016$ in each period.¹⁹ In the baseline model, an inflationary Phillips curve shock accumulates output gains through ten quarters by 15.3%, which is the same as when the minimum wage is increased. However, with hand-to-mouth households these gains are substantially less than the equivalent minimum wage increase: 20.4% rather than 24.1%.

This result further highlights the relationship between minimum wages and the Paradox of Toil. As shown in Section (3.1), accounting for the minimum wage extends the Paradox of Toil mechanism to economies that could not be previously studied with this model – those in which the ZLB episode is expected to be highly persistent. That increasing the minimum wage is more expansionary than purely inflationary shocks to the Phillips Curve in the model with hand-to-mouth consumption suggests that it may be a more effective, if more unconventional, policy instrument during future ZLB episodes.

4 Conclusion

The minimum wage is a perennial topic of policy debate and the sticky-price New Keynesian model provides a natural framework to study its aggregate effects. I have presented a parsimonious extension of the textbook model and draws a stark conclusion: the aggregate effect of increasing the minimum wage is dictated by the state of the economy and monetary policy’s response to inflation. In normal times, the central bank’s response to inflation determines the effect of increasing the minimum wage: if monetary policy responds according to a standard Taylor Rule then increasing the minimum wage is contractionary. However, when interest rates are stuck at zero, increasing the minimum wage raises inflation, but does not instigate a countervailing monetary response; it

¹⁹This ensures that the initial shock to the Phillips curve is the same magnitude as when the minimum wage is increased by 41.8%. Note that the paths of output and inflation gains are the same in the baseline model in Figures (9) and (10).

is therefore expansionary.

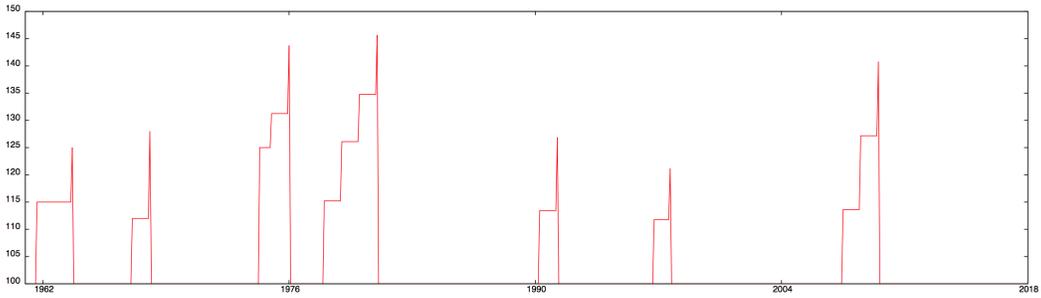
Even if the minimum wage is constant, introducing it into the model reshapes the economy's response to macroeconomic shocks. The minimum wage greatly reduces the severity of a demand-driven ZLB episode by dampening the deflationary effects of such demand shocks, thereby halting the deflationary spiral associated with the zero lower bound. While the share of minimum wage workers determines the extent to which the ZLB's ill effects are alleviated, any positive share of these workers has an infinitely positive effect in the face of a sufficiently persistent ZLB shock. Happily, an economy with a realistically small share of minimum-wage earners reduces the negative effects of the ZLB by more than half.

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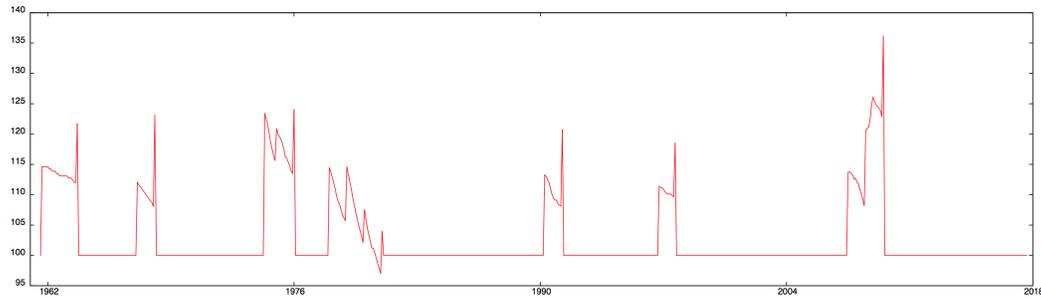
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(a) Nominal Min. Wage



(b) Real Min. Wage

Each increase in the nominal minimum wage covers those planned upon amendment of the Fair Labor Standards Act of 1938. Plots are rolling indices, so return to 100 in the month following the last legislated increase for the corresponding amendment. Many amendments prior to the 1960s affected coverage as well as the nominal minimum. The top panel shows that the 1978-1981 increases were the largest in nominal terms, while the bottom panel shows that the 2007-2009 increases were the largest in real terms.

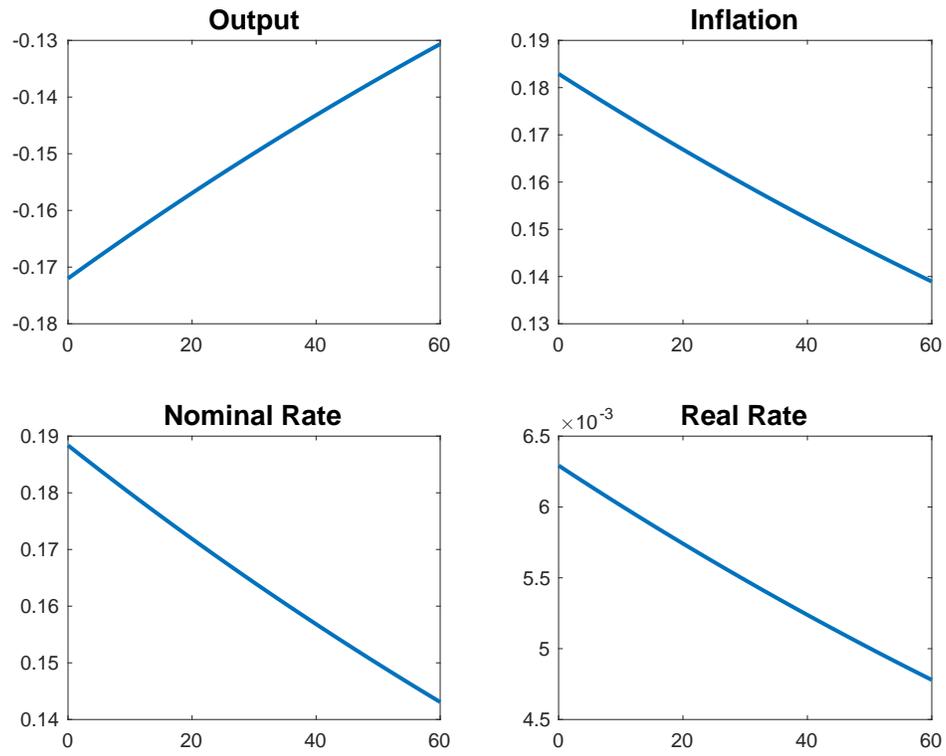


Figure 1: Aggregate Responses: Contractionary Monetary Response

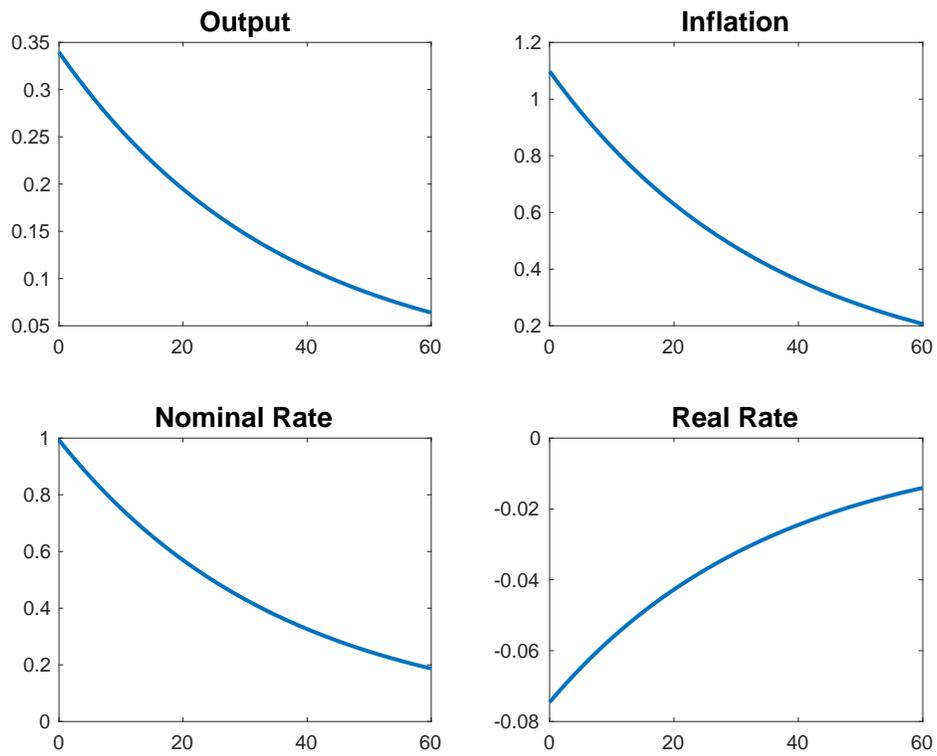


Figure 2: Aggregate Responses: Expansionary Monetary Response

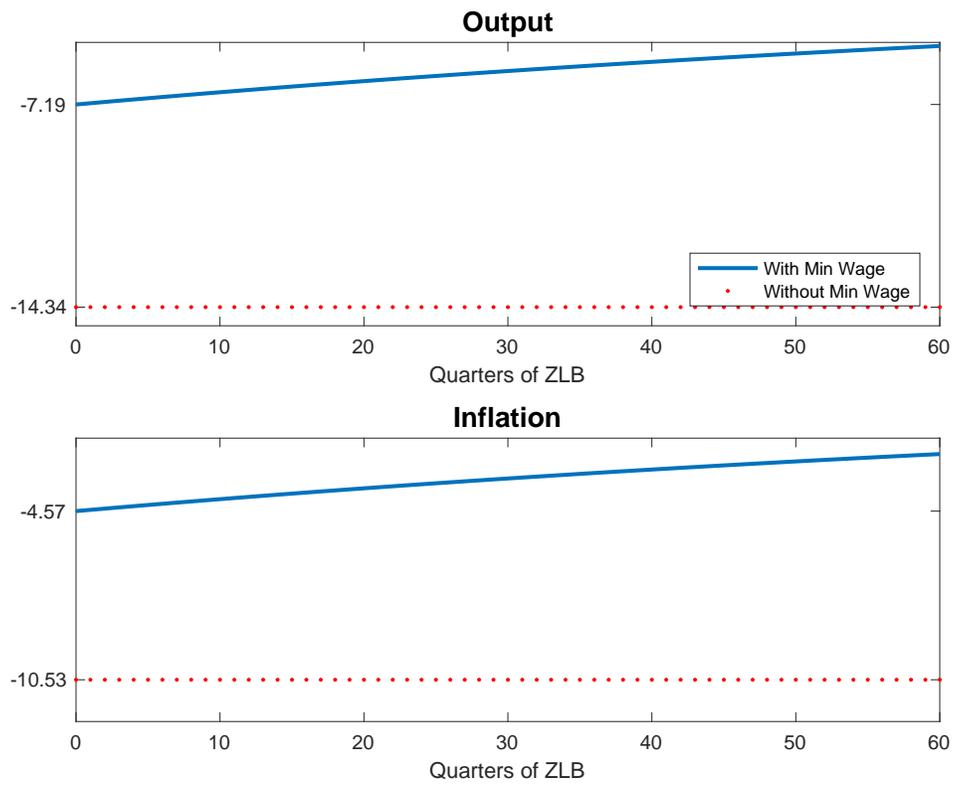


Figure 3: Aggregate Effect of ZLB Shock

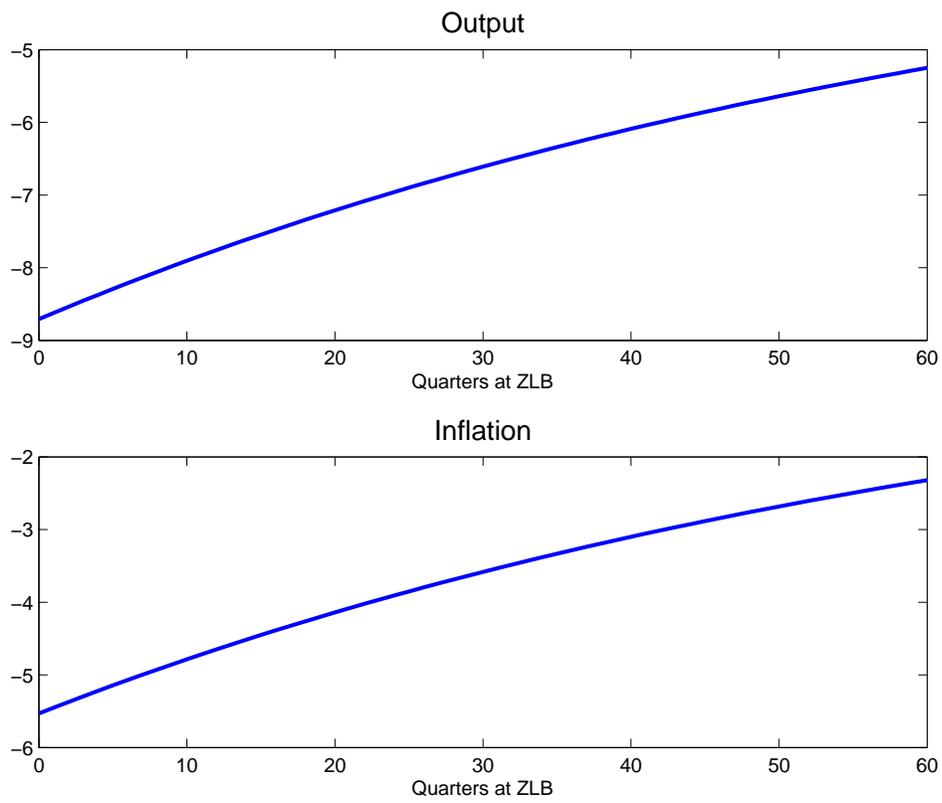


Figure 4: Aggregate Effect of ZLB Shock: $\chi = 0.9091$

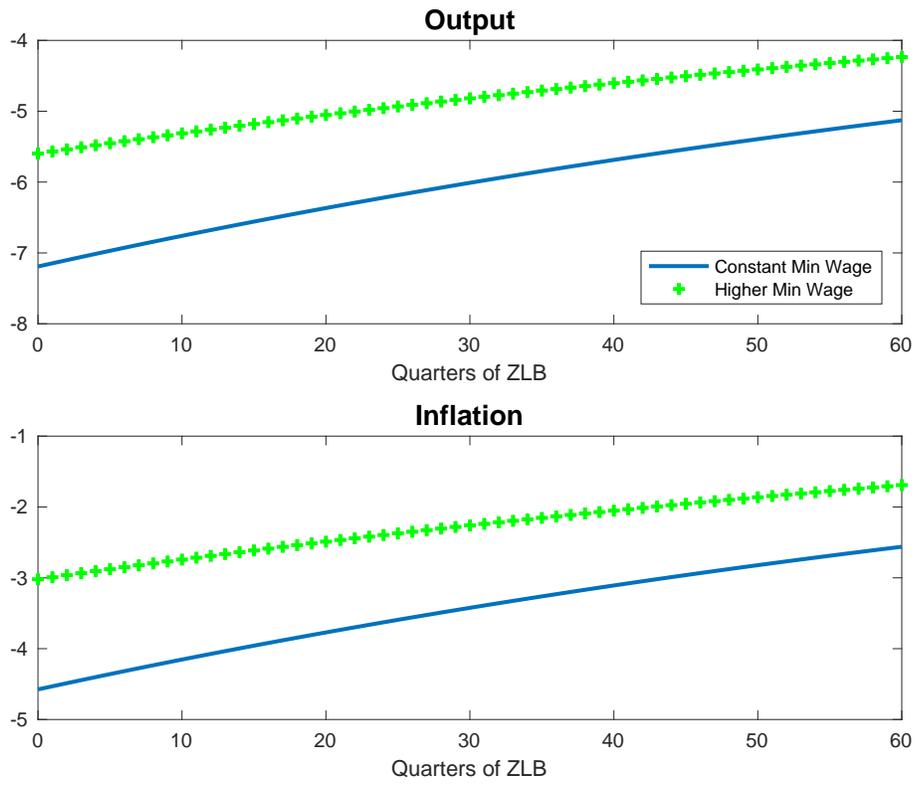


Figure 5: Effect of Increasing Min Wage During ZLB

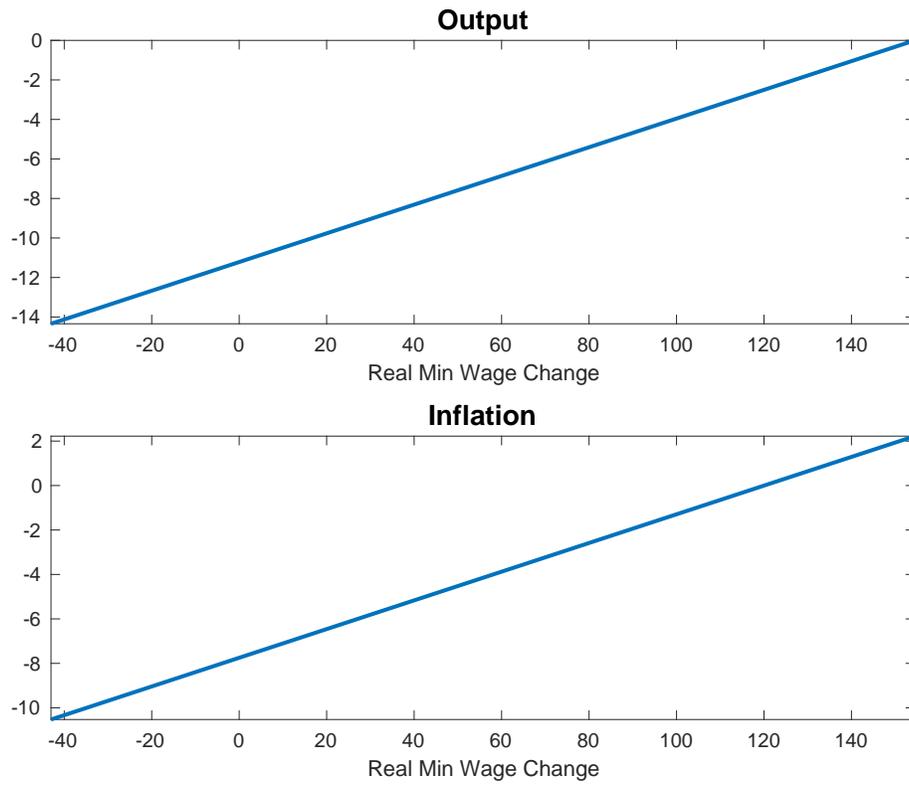


Figure 6: Effect of Indexing Min Wage During ZLB

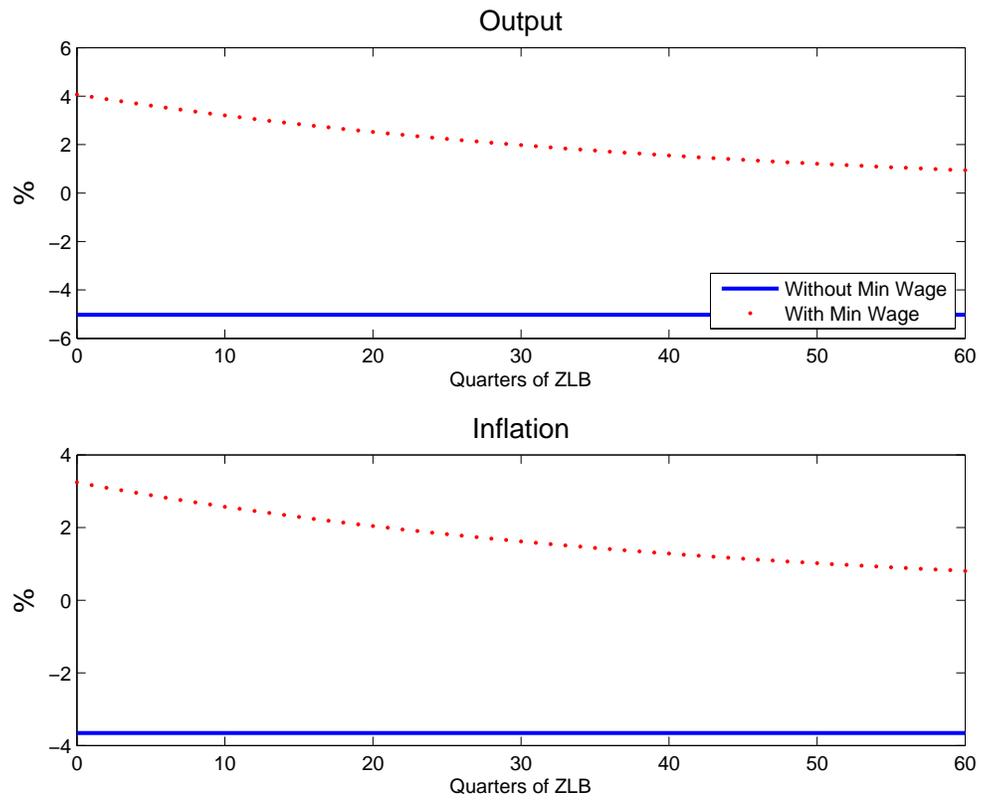


Figure 7: Gains From Inflationary Policy, Highly Persistent ZLB Shock

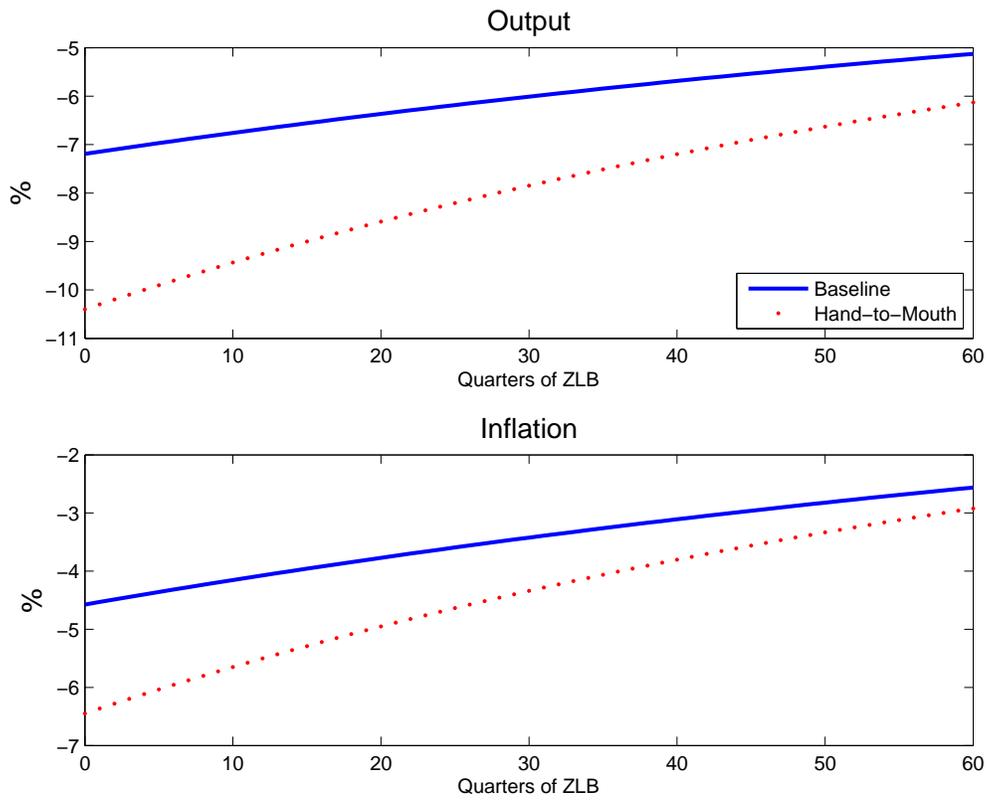


Figure 8: Aggregate Effect of ZLB Shock: Hand-to-Mouth Households

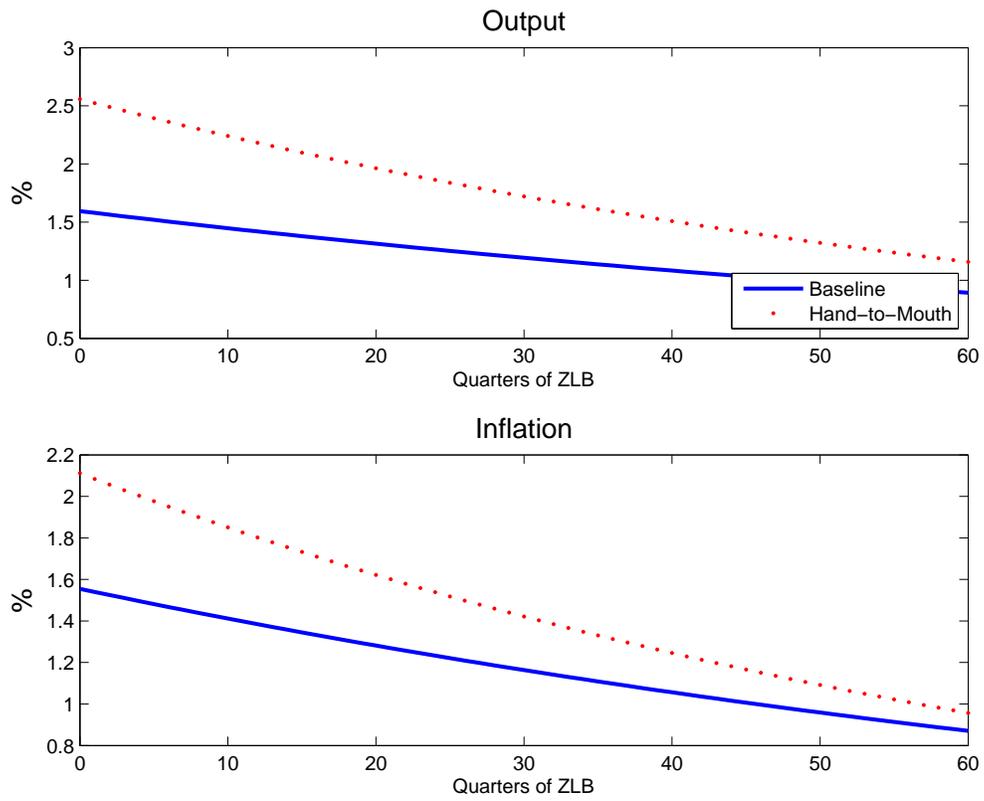


Figure 9: Gains From Raising Min. Wage During ZLB: Baseline Vs. Hand-to-Mouth

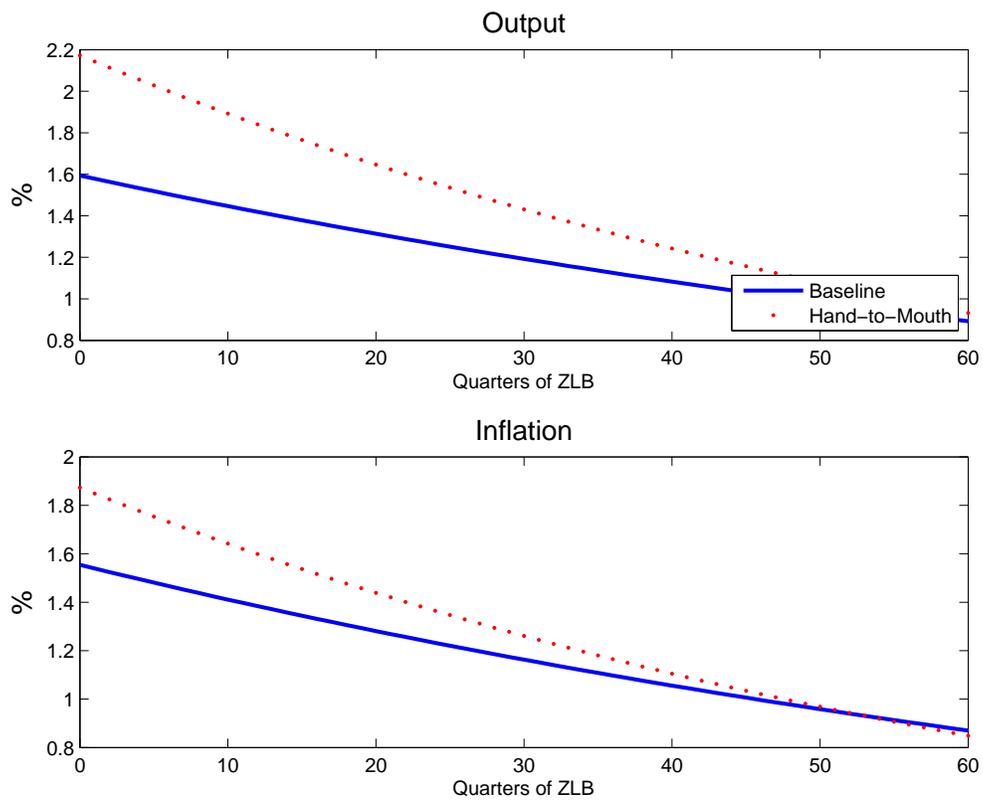


Figure 10: Gains From Inflationary Policy During ZLB: Baseline Vs. Hand-to-Mouth

A Appendix

The system describing steady-state is as follows:

$$1 + i = \beta^{-1} \quad (39)$$

$$N_H = \left(\Psi^{-1} \frac{W_H}{P} Y^{-\sigma} \right)^\nu \quad (40)$$

$$N_L = \min \left\{ \frac{\epsilon_H}{\epsilon_L} N_H \left(\frac{W_L}{\theta W_H} \right)^\eta, \left(\Psi^{-1} \frac{W_L}{P} Y^{-\sigma} \right)^\nu \right\} \quad (41)$$

$$\frac{\epsilon_L N_L}{\epsilon_H N_H} = \left(\frac{W_L}{\theta W_H} \right)^\eta \quad (42)$$

$$Y = \left((\epsilon_H N_H)^{\frac{\eta-1}{\eta}} + \theta (\epsilon_L N_L)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \quad (43)$$

$$\frac{\mathcal{W}}{P} = \gamma^{-1} \quad (44)$$

$$\frac{\mathcal{W}}{P} = \left(\left(\frac{\theta W_H}{W_L} \right)^{\eta-1} + \theta \right)^{\frac{\eta}{\eta-1}} \frac{W_L}{P} \quad (45)$$

The steady-state with a just-binding real minimum wage is the starting point for analysis. I now derive the four equation model and the expressions for real wages, hours and earnings. To begin I list the linearized equilibrium

conditions:

$$y_t = \mathbb{E}_t y_{t+1} - \sigma^{-1}(i_t - r_t - \mathbb{E}_t \pi_{t+1}) \quad (46)$$

$$i_t = \max\{0, r_t + \psi_y y_t + \psi_\pi \pi_t\} \quad (47)$$

$$n_{H,t} = \nu(w_{H,t} - p_t) - \sigma \nu y_t \quad (48)$$

$$\phi_t = \eta(w_{H,t} - w_{L,t}) \quad (49)$$

$$n_{L,t} = \phi_t + n_{H,t} \quad (50)$$

$$n_{H,t} + s_L \phi_t = \delta_t + y_t \quad (51)$$

$$\delta_t = 0 \quad (52)$$

$$\omega_t = w_{L,t} - \mu \phi_t \quad (53)$$

$$p_t^\# = (1 - \beta\lambda)\omega_t + \beta\lambda\mathbb{E}_t p_{t+1}^\# \quad (54)$$

$$p_t = \lambda p_{t-1} + (1 - \lambda)p_t^\# \quad (55)$$

$$\pi_t = p_t - p_{t-1} \quad (56)$$

Where all variables are in percentage deviations from steady state except for those already expressed as rates, which are in absolute deviations. The new parameter μ is the elasticity of the marginal product of low-productivity workers with respect to their labor input evaluated in steady state. With the constant elasticity production function this parameter is constant and can be written as $\mu = \eta^{-1}(s_L - 1)$. The intermediate variable ϕ_t is the ratio of low to high productivity workers. Combining Equations (48), (49), and (51) gives the high-productivity real wage as a function of output and the real minimum wage:

$$w_{H,t} - p_t = \frac{1 + \sigma\nu}{\eta s_L + \nu} y_t + \frac{\eta s_L}{\eta s_L + \nu} (w_{L,t} - p_t) \quad (57)$$

This then gives ϕ_t as:

$$\phi_t = \frac{\eta(1 + \sigma\nu)}{\eta s_L + \nu} y_t - \frac{\eta\nu}{\eta s_L + \nu} (w_{L,t} - p_t) \quad (58)$$

Which then gives the real unit cost of production as:

$$\omega_t - p_t = \left(1 + \mu \frac{\eta\nu}{\eta s_L + \nu}\right)(w_{L,t} - p_t) - \mu \frac{\eta(1 + \sigma\nu)}{\eta s_L + \nu} y_t \quad (59)$$

And finally, setting $\mu = \eta^{-1}(s_L - 1)$ and using Equations (54), (55), and (56) gives the Phillips Curve as in Equation (27). The labor supply and real earnings of each worker type are then recovered from the appropriate equations above.