The Life-Cycle Distribution of Earnings and the Decline in Labor’s Share

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Abstract

We estimate the effect of the life-cycle distribution of earnings on labor’s share of income. We relax the assumption of perfectly competitive wages and show that the aggregate labor share is no longer a simple function of production parameters, but is instead an earnings-share-weighted harmonic mean of labor shares across demographic groups. We document that the share of earnings accruing to elder workers has risen sharply in recent years, coincidental with the majority of the decline in labor’s share. We then use an IV approach to estimate that a one percentage point shift in earnings towards elder workers leads to a 0.29 percentage point decline in labor’s share. We rationalize our empirical findings by extending two standard theories of frictional labor markets to include a life-cycle of productivity which endogenously grows faster than earnings.
1 Introduction

In this paper, we relax the assumption that a worker’s wage is set under perfect competition and estimate the effect of the age-distribution of earnings on aggregate labor’s share. We derive a novel accounting identity which links labor’s share and the relative earnings wedges\(^1\) of workers across demographic groups. We document that a sharp increase in the earnings shares of elder workers (aged over fifty years) has occurred alongside a large decline in labor’s share since the late nineties. We estimate the relative earnings wedges between young and elder workers and find that elder workers receive about \(3/4\) of their marginal product as earnings relative to younger workers. With our estimate of the relative earnings wedge, we predict that labor’s share would have barely changed since the late nineties if not for the rise in the earnings of elder workers.

In order to understand how we identify this estimate, consider two increases in labor supply with differential effects on output and earnings. First, one day we find an exogenous one hour increase in the supply of youth labor, followed by a $1 increase in output and the same increase in total labor income. The next day we observe an exogenous one-hour increase in elder labor supply along with a $1 increase in output but only an $0.71 increase in earnings. From this we would infer that elder workers, while equally productive as the young, are able to capture only 75% of their marginal product relative to the young.\(^2\)

Our estimation procedure is quite general and is valid for many theories which may generate life-cycle heterogeneity in the earnings wedge. Specifically, we use population shares as an instrument for exogenous shifts in earnings share. The two series are extremely strongly correlated and we argue that population shares are likely exogenous: all that we require is for twenty-year lagged birth rates and contemporaneous mortality rates to be uncorrelated with current shocks to labor’s share.

We also provide evidence from regional and sectoral movements in labor share and earnings shares. The results are largely in line with the aggregate data, although we must use alternative instruments for labor supply since population shares are no longer valid (for sectors they are not even defined). We use a common Bartik style instrument and find that almost all regions have similar relative earnings wedges to the United States.

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\(^1\)As defined formally below, the earnings wedge is the inverse of the fraction of a worker’s marginal product paid as salary. A higher wedge means that a worker receives a smaller share of his marginal product as earnings.

\(^2\)For this example we assume that the young earn exactly their marginal product, but this need not be true. We also assumed that both youth and elder labor had the same marginal product, but the logic would be unchanged if they differed.
States, as do two-thirds of sectors for which we have a valid instrument.

The factors determining labor’s share and its relation to the aggregate production function were once closed questions, but are now very much open. From Kaldor ([2]) to very recently, macroeconomists typically assumed a constant labor’s share of GDP. This assumption, along with the assumption that wages are set in competitive spot markets, restricts the aggregate production function to the Cobb-Douglas form and exactly identifies the elasticity of output with respect to labor. This elasticity is labor’s share of GDP. Recent research \(^3\) indicates that the assumption has been unrealistic since the early 1980s and that it has become even less appropriate for the United States since the early 2000’s. This decline has coincided with a renewed concern over the distribution of income between factors (Piketty ([8] for example) and the question of how policy should influence the distribution of income.

The existing literature typically maintains a neoclassical aggregate production function along with perfectly competitive labor markets and relies upon shocks to explain variation in labor’s share. Karabarbounis and Neiman ([3]), estimate a CES production function on international data. Their estimates imply that capital and labor are more substitutable than one (the Cobb-Douglas case), so that a decline in the price of investment goods generates a rise in the capital labor ratio and a fall in labor’s share. In a recent working paper, Lawrence ([5]) estimates a similar production function but finds just the opposite of Karabarbounis and Neiman - he estimates that the elasticity of substitution is less than one, but that the productivity weighted capital labor ratio has actually fallen over the time period in which labor’s share has declined.

Rather than vary the aggregate production technology and attempt to measure the appropriate capital-labor ratios, we relax the assumption that a worker’s wage is set under perfect competition. We are not the first to consider non-competitive factors as a cause of labor share’s decline. The paper closest to ours is Elsby, et al ([1]), who use industrial data on import competition and labor share. They find that industries which experienced larger increases in import competition also experienced larger declines in labor share. \(^4\) We find a similar result using the same industrial data - industries for which elder workers experienced the most earnings growth tend to have larger declines in labor’s share.

We focus on a common class of models with labor market search frictions. In these

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\(^3\) The literature is growing. Karabarbounis and Neiman have provided an overview and their research agenda ([4]).

\(^4\) They also argue against capital deepening due to lower investment prices, since there is no relationship between changes in industry level investment goods prices and labor share.
models, the worker-firm relationship generates a match surplus which must be split according to some protocol. We show that the work-horse Diamond-Mortensen-Pissarides ([6]) can generate a rising earnings wedge over the life cycle due to horizon effects. That is, an elder worker has a shorter expected duration, and therefore a smaller match surplus (ceteris paribus). His earnings therefore amount to a smaller fraction of his marginal product. Crucially, this can hold even if he is more productive and therefore receives a higher level of earnings.

We also consider an extension to the model by Postel-Vinay and Robin ([9]) in which workers search on the job and employers compete ala Bertrand when two are in contact with the same worker. In this model, workers receive none of the surplus until they receive an outside offer, at which point they receive the minimum of the surplus from the two competing firms (manifest as an increase in earnings). To this model we add growth in match specific productivity. Outside offers are then met with a wage increase, but only to the point that exhausts the surplus of a new (lower productivity) match. This creates an expanding gap between earnings and marginal product as a worker continues on a job, which translates to an increasing life-cycle profile of the earnings wedge.

2 Accounting For Aggregate Labor’s Share

We proceed with three basic assumptions which allow for an estimatable accounting identity for labor’s share. Our first assumption relaxes the assumption that workers are paid 100% of labor’s marginal product. Specifically, we will assume that at any date $t$, for each worker $i = 1, ... I_t$, we have the following equation for earnings:

$$ e_{i,t} = \frac{1}{\alpha_i + \epsilon_t \partial Y_{i,t}} $$

The parameter $\alpha_i$ reflects an earnings wedge between the worker’s pay and his marginal product. Specifically, a larger wedge means that the worker receives a smaller fraction of his marginal product. This parameter will assume to be constant for a given worker (or in practice a a grouping of workers) over time, while the error term $\epsilon_t$ may vary (but is identical across workers). We will make weak assumptions on the error term below.

Our second assumption is that the aggregate elasticity of output with respect to all labor inputs is constant (or at least over the period of estimation). This allows us to link the sum of marginal products to aggregate output, which will be then allow us to relate output and total earnings using the earnings wedge assumption above. Specifically, we
assume that:

\[
\sum_{i=1}^{I_t} \frac{\partial Y_t}{\partial n_{i,t}} = \alpha Y_t
\]

If this assumption holds at all times, then it implies that the aggregate production function has unit elasticity of substitution between capital and some constant returns aggregator of labor. Alternatively, it would hold for any production function exhibiting constant returns to scale over any time period in which an appropriately defined capital-labor ratio is constant. \(^5\)

Finally, we will assume that it is possible to group workers in such a way that they share the same \(\alpha_i\) parameter. That is, we can find sets \((D_j)_{j=1}^J\) such that \(\forall t \geq 0, \forall i = 1, 2, \ldots, I_t,\) there is exactly one \(D_j\) such that \(i \in D_j\) and for every \(i, i' \in D_j\) we have that \(\alpha_i = \alpha_{i'} = \alpha_j.\)

With these three assumptions, we derive our main accounting identity. For each \(j\) and each \(i \in D_j,\) we can rearrange \(\frac{\partial Y_t}{\partial n_{i,t}} = \alpha_j^{-1} e_{i,t}.\) Notice that the marginal product on the left-hand side is unobservable, as is the wedge term on the right hand side. However, if we define \(E_{j,t} \equiv \sum_{i \in D_j} e_{i,t} n_{i,t}\) and sum over all of the groups, then we have:

\[
\alpha Y_t = \sum_{j=1}^J \alpha_j E_{j,t} + \epsilon_t \sum_{j=1}^J E_{j,t}
\]

And finally dividing by \(E_t \equiv \sum_{j,t} E_{j,t}\) and defining the earning share of group \(j\) as \(\sigma_{j,t} = \frac{E_{j,t}}{E_t},\) we arrive at the accounting identity:

\[
\frac{1}{LS_t} = \sum_{j=1}^J \alpha_j^{-1} \sigma_{j,t} + \epsilon_t
\]

This is a simple accounting identity which rests on relatively few structural assumptions. However, there are two items for which we must rely on theory. The first is the grouping of households by earnings wedge. Any deviation from perfectly competitive wages requires a theory of how workers and employers split the surplus from their relationship and a grouping of workers by wedges relies upon a systematic variation in this split across groups. We develop a theory for which age is the natural dimension

\(^5\)If we don’t assume a constant elasticity, then we would have \(\sum_{i=1}^{I_t} \frac{\partial Y_t}{\partial n_{i,t}} = Y_t - \frac{\partial Y_t}{\partial K_t} K_t = Y_t \left(1 - \frac{\partial Y_t}{\partial K_t} \frac{K_t}{Y_t}\right)\). With constant returns to scale and constant capital-labor ratios across all groups, the term in the final parenthesis is constant.
along which wedges differ. The second item is the residual $\epsilon_t$, which is of paramount importance for consistently estimating relative wedges for a given grouping of workers. We will specifically worry about the correlation of $\epsilon_t$ since shocks to technology may simultaneously drive down labor’s share and shift earnings towards elder workers.

Finally, we note that this equation holds for any level of aggregation for which the production function has a constant elasticity $\alpha$. We will provide evidence from regional and industrial level below, but this caveat (along with others) will limit our ability to interpret the results structurally.

3 Data

Our measure of aggregate labor share comes from the Bureau of Labor Statistics at the state level, which we then aggregate to compute the national labor share. This has the same dynamics as the index provided by the Federal Reserve Bank of St. Louis, which is readily available. We use annual data from the March Current Population Survey to get earnings shares by group for both the aggregate economy and regions of the united states. We also get population shares by group, which will be used as an instrument in the estimation to follow. This introduces some inconsistency between our earnings and those used for labor’s share, since we cannot account for benefits, whereas the BLS includes them in the labor’s share computations. As long as there is no consistent difference in the fraction of compensation due to benefits across groups, this will not affect the results. Our industrial level labor share data is provided by Elsby, et al ([1]) and is publicly available through the Brookings website.

Figure 1 shows the aggregate labor share from 1962 to 2013. The period from 1962–2000 exhibits a slight downward trend, but also extended periods of increase. As recently as 2000 labor’s share was at 0.64, which is essentially the average value through 1980. The stark change begins right after 2000, when labor’s share begins to decline and hasn’t experienced robust increases for any amount of time since. By the end of the sample, in 2013, labor’s share has fallen from 0.64 to just below 0.58.

We have only 51 observations for labor’s share, which limits how finely we can group the population. Our baseline grouping is purely based on age. We split the population into five age groups, the youngest group being 17 to 29 years and the oldest group consisting of 60 years and older. For households with business or farm income, we increase earnings by a proportion of business or farm income consistent with the aggregate labor
share.  

For our baseline case, Table 1 summarizes the earnings shares and Figure 2 plots the time series for earnings shares relative to their means. Figure 2 shows that the eldest two age groups have experienced dramatic increases in their earnings shares. The group aged over sixty years has seen their share more than double and workers in their fifties have seen their share substantially as well.

4 Accounting Results

Equation 4 can be estimated using linear methods once we specify properties of the residual $\epsilon_t$. That is, we will estimate the following regression:

$$LS_t^{-1} = \beta_1 + \sum_{j=2}^{J} \beta_j \sigma_j + \epsilon_t$$  \hspace{1cm} (5)

For different assumptions on $\epsilon_t$. Since the earnings shares sum to one, we have dropped the first group into the constant, we will then use the estimates to find the relative earnings wedges for groups $j \geq 2$ via:

$$\frac{\hat{\alpha}_j}{\hat{\alpha}_1} = \frac{\hat{\beta}_j + \hat{\beta}_1}{\hat{\beta}_1}$$

4.1 Groupings and the Residual

We must first choose the demographic groupings which determine $J$. Our baseline results will use just two age groups, with $j = 1$ corresponding to workers younger than 50 and $j = J = 2$ corresponding to those 50 and up. We also estimate the model with a finer age grouping but find that the split between young and elder is most important.

We must also address the residual $\epsilon_t$ in Equation (4). We estimate Equation (4) under different assumptions. First, we will assume that $\epsilon_t$ is purely aggregate without a trend and estimate the following model using ordinary least squares, allowing for serial correlation in the residual:

$$LS_t^{-1} = \beta_0 + \beta_1 \sigma_{elder,t} + \epsilon_t$$  \hspace{1cm} (6)

We perform robustness with respect to this assumption in the appendix, since Elsby, et al ([1]) have shown that the decline in aggregate labor’s share is somewhat sensitive to the handling of business income.
These estimates are seen in Column (1) of Table (2). The coefficient on elder-earnings share, $\beta_1$, is significantly positive, which indicates that elder workers have a larger earnings wedge than young. We next allow for a trend in the residual term and estimate the following model with Ordinary Least Squares:

$$LS_t = \beta_0 + \beta_1 \sigma_{elder,t} + \gamma t + \epsilon_t$$

(7)

Where $t$ is defined as the current year minus 1963, which is the first year in our sample. The estimates from this model are reported in Column (2) of Table (2). The estimate of $\beta_1$ is still significantly positive, though it has fallen from 0.79 to 0.52. While the labor share and earnings shares are, in principle, stationary, we prefer to allow for a time trend using this specification.

If the error term $\epsilon_t$ is not purely aggregate, but instead represents a shock that both reduces labor share and shifts earnings shares, then the above models will not give a consistent estimate of the relative earnings wedges. Our preferred specification allows for these shocks and instruments the elder-earnings share with the elder-population share. So long as 20-year lagged birth rates and contemporary mortality rates are uncorrelated with the residual in Equation (7) then population share is an exogenous variable. It is clearly strongly correlated with earnings shares, as can be seen in Figure (10) and the F-statistic indicates it is a strong instrument, so is analogous to a factor that exogenously shifts the supply of elder labor. The results of this estimation can be found in Column (3) of Table (2). The coefficient on elder-earnings share has risen to 0.57 which, along with the estimated constant, implies that elder workers earn 71% of their marginal product relative to young workers, as can be seen in Column (4).

The final column in Table (2) reports the IV estimation in which all variables are in first differences. While our variables are stationary, in principle, and we think that the coincidental timing of the decline in labor's share and rise in elder-earnings share is an important feature of the data, we present this estimate as robustness. The coefficient is large in sign, though at the margin of significance.

The overall fit of this regression can be seen in Figure (11); the predicted labor share tracks the data quite closely, though with such a parsimonious model we should cannot match every short-run movement. Using the estimates from our preferred specification, we plot the actual labor share time series, the predicted, and a counterfactual series in which we set the earnings share of elder workers equal to the initial value. These plots can be seen in Figure (12), which shows that the dynamics of labor's share would have
differed dramatically if earnings shares had remained constant. Specifically, we predict that the decline in labor-share since 2000 would have been substantially muted if not for the shift in earnings towards elder workers.

We now look at the aggregate data at a finer age resolution. It is instructive to consider the underlying factors which determine earnings shares. Specifically, the share of labor in each age group and the relative earnings-per-worker in each age group. We use the identities for this exercise:

$$E_{i,t} = \omega_{i,t} N_{i,t}$$

Where $$\omega_{i,t}$$ is average earnings per person in group $$i$$. Then group-specific earnings shares can be written as:

$$\sigma_{i,t} = \frac{\omega_{i,t} N_{i,t}}{\omega_t N_t}$$

Where we have defined $$\omega_t = \frac{E_t}{N_t}$$. We now define $$\sigma_{i,t}^\omega = \frac{\omega_{i,t}}{\omega_t}$$ and $$\sigma_{i,t}^N = \frac{N_{i,t}}{N_t}$$, which gives the decomposition plotted for each age group in Figure 4 through Figure 8.

$$\log \sigma_{i,t} = \log (\sigma_{i,t}^\omega) + \log (\sigma_{i,t}^N)$$

We can understand the changes in earnings shares through these decompositions. The growth in the share of aggregate earnings going to group $$i$$ is just the growth in that group’s average earnings per person relative to the economy wide earnings per person plus the growth in that group’s workforce size relative to the total workforce. We first document the decomposition of each cohort’s earnings share and then use these to perform more detailed counterfactuals.

The decompositions highlight that the rise in the earnings share of elderly households is not purely due to the increase in the raw number of baby boomers. This can be seen in Figure 8, where we decompose the earnings share for households over the age of 60. The rise in this group’s earnings share since 2000 has been mostly due to a rise in their earnings-per-worker relative to the economy wide average. Only recently has the share of total workers in this group started to rise. However, the 50-59 year old group owes most of its increase in earnings share to an increase in the relative number of these workers rather than in their earnings-per-worker.

On the other side of the life-cycle, the share of earnings for the youngest households (below age 30) can be seen in Figure 4. Here we see that the share of total workers drove the decline of this group’s earnings share from the early 80s through mid-90s, but that
their relative earnings have declined since then.

4.2 Sectoral Variation

While we have written Equation 4 at the national level, it can also be used at any finer level of aggregation for which labor (or payroll) shares and group-specific earnings shares can be calculated. In this section we estimate the empirical model using sectoral data. This is an especially attractive level of aggregation since the national (or any geographic aggregate) labor share may vary over time due to changing sectoral composition of value added if sectors use technologies with vastly different labor elasticities.

One concern at the sector level is finding an instrument, since workers of different ages may relocate across sectors in response to shocks that impact earnings differentially. We obviously cannot use population shares as an instrument, therefore we use a common alternative originally proposed by Bartik (CITE). We instrument the elder earnings share in a given region or sector with the average earnings-share of all other regions or sectors. We use these instruments to estimate two specifications of the model: a pooled fixed-effects regression in first differences and sector-by-sector with time trends (which allows for all parameters to differ freely across sectors).

4.3 Sectoral Labor Share

We follow Elsby, et al ([1]) to construct sectoral payroll shares from 1987 and 2011, which includes the post-2000 period during which the aggregate labor share fell most sharply. We construct the earnings shares at the industry level using CPS industry codes and match earnings and payroll shares for 11 sectors (3). The average number of observations in the CPS for a sector-year pair is 7,016. Thus, for each sector we have the payroll share, the earnings shares of individuals under and over 50 years of age, and the national earnings share of individuals over 50 years of age. Our sectoral analysis focuses on two specifications: First, using the panel of sectors to analyze a version of equation (4) in first differences in which we include year fixed effects. Specifically,

$$
\Delta PS_{s,t}^{-1} = \beta_1 \Delta \left[ \frac{E_{s,elder,t}}{E_s,t} \right] + \psi_t + \eta_{s,t} 
$$

Since workers can change sectors population shares of mature workers are no longer valid instruments for the earnings share. Therefore, following Nakamura and Steinsson [7], we use the national earnings share of individuals over 50 to instrument for the
earnings share of elder workers within a sector, \( \frac{E_{s,elder}}{E_s} \). Including the year fixed effects implies that identification of \( \beta_1 \) comes from the cross-sectoral differences in the response of payroll shares (relative to the average) to changes in the sectoral earnings share of mature workers. Table 4 shows the results from the panel regression, the coefficient on the earnings share of mature workers is positive and significant. In terms of the relative earnings wedges for each sector, the above specification imposes the following restriction:

\[
\beta_1 = \alpha_s^{-1}(\alpha_{s,elder} - \alpha_{s,young})
\]

for all sectors. Therefore, the positive coefficient in table 4 suggests that mature workers have a larger earnings wedge and receive a smaller portion of their marginal product, consistent with the findings at the national level.

The second specification we analyze is to impose that equation (7) holds at the sectoral level,

\[
P S_{s,t}^{-1} = \beta_{s,0} + \beta_{s,1}\sigma_{s,elder,t} + \gamma_s t + \epsilon_{s,t}
\]

(9)

where the national earnings share of elder workers is used to instrument for the sector earnings share. The results from the sector-by-sector regressions are shown in Figure 13. At the sectoral level the relative wedge estimates are much less precise and the instruments tend to be weaker\(^7\). However, most point estimates are below one, indicating a larger earnings wedge for elder workers relative to the young.

5 Theories of Life-Cycle Earnings Wedges

Frictional models of the labor market are natural candidates for explaining the difference in earnings wedges across age groups. We demonstrate how two different bargaining models can endogenously generate earnings that grow slower than productivity. Conceptually, there are three reasons: the match surplus may decline with age, the arrival rate of credible outside offer rates may fall, and a worker’s productivity may become more match specific over time. We illustrate these points qualitatively by extending two common search and matching models to include a life-cycle profile of productivity and other payoff relevant parameters.

\(^7\)Since the national earnings share was a very weak instrument for Natural Resources and Mining and Leisure and Hospitality, estimates from these sectors are not reported.
5.1 Life Cycle DMP Model

We first extend the Diamond-Mortensen-Pissarides model to allow for aging and retirement. With probability $\rho < 1$, an elder household transitions into retirement. With probability $\eta$ a young household transitions into the elder state. Households are risk neutral and have discount factor $\delta$. A household may be in one of five states. He begins in the young age group and unemployed. He finds a job with an endogenous probability $f(\theta_j)$ for $j = young(y), elderly(o)$, at which point he becomes a worker. As a worker, he faces an exogenous probability of separation, $s$, into unemployment. The probability of separation is assumed to be the same for both ages. Aging occurs with exogenous probability $1 - \eta$. In addition, an elderly worker retires at the end of each period with probability $\rho$. Retirement is an absorbing state and gives a value normalized to zero. The value functions for a household are given by:

$$W_y = e_y + \delta [(1 - \eta) ((1 - s)W_y + sU_y) + \eta ((1 - s)W_o + sU_o)]$$  (10)

$$W_o = e_o + \delta (1 - \rho) [(1 - s)W_o + sU_o]$$  (11)

$$U_y = z + \delta [(1 - \eta) (f_y W_y + (1 - f_y)U_y) + \eta (f_o W_o + (1 - f_o)U_o)]$$  (12)

$$U_o = z + \delta (1 - \rho) [f_o W_o + (1 - f_o)U_o]$$  (13)

where $e_j$ for $j \in \{y,o\}$ are the earnings of the young and old, and $z$ represents the unemployment benefits accrued in the unemployed state.

Recall that the young and old are assumed to differ in two respects. First, the young have a longer expected job duration than the old, since they must transition through the old state before retirement can occur. Second, they are assumed to have different marginal products when employed by the firm. These two differences will be key determinants of the difference in their endogenous earnings, $e_y$ and $e_o$, which is described below.

There is a large number of firms who may post vacancies in order to hire workers. Posting a vacancy, conditional on age, leads a firm to meet a worker with probability $q(\theta_j)$ for $j = y, o$. Free entry in each market, $j$, guarantees that the market tightness, $\theta_j$, adjusts so that the discounted expected profits from posting a vacancy equate to the cost, $\kappa$. A firm that employs a young worker generates revenue $p_y$ while a firm that employs an elderly worker generates revenue $p_o$. This gives firm value functions and
Finally we make the assumption that wages are determined through Generalized Nash Bargaining in which the worker has bargaining parameter $\beta \in (0, 1)$, which itself is age-invariant:

$$W_y - U_y = \beta [W_y + \Pi_y - U_y] \tag{18}$$
$$W_o - U_o = \beta [W_o + \Pi_o - U_o] \tag{19}$$

Using these equations, it is straightforward to derive the earnings of both young and old workers:

$$e_y = (1 - \beta)z + \beta p_y + \beta \kappa ((1 - \eta)\theta_y + \eta \theta_o) \tag{20}$$
$$e_o = (1 - \beta)z + \beta p_o + \beta \kappa (1 - \rho)\theta_o \tag{21}$$

Note that the earnings of the young and old reflect both the difference in their marginal products, as well as, a share of future surplus. Reflecting the short time horizon before retirement, the share of future surplus will be smaller for the elderly worker.

### 5.2 Steady State and Calibration

Given exogenous fractions of young, $N_y$, and old, $N_o$, households, we can use the law of motion of employment to compute the steady state employment rates. The law of motions of employment are:

$$\text{EMP}'_y = (1 - s)(1 - \eta)\text{EMP}_y + f_y(N_y - \text{EMP}_y)$$
$$\text{EMP}'_o = (1 - s)[\eta \text{EMP}_y + (1 - \rho)\text{EMP}_o] + f_o(N_o - \text{EMP}_o)$$
which imply employment rates for the young and old:

\[
\begin{align*}
\frac{\text{EMP}_y}{N_y} &= \frac{f_y}{1 + f_y - (1 - s)(1 - \eta)} \\
\frac{\text{EMP}_o}{N_o} &= \frac{(1 - s)\eta \frac{\text{EMP}_y}{N_y} N_y + f_o}{1 + f_o - (1 - s)(1 - \rho)}
\end{align*}
\]

We calibrate the model to the labor market in 2000 and use the Hagedorn and Manovskii (2008) strategy to pin down $z$ and $\beta$. Table 5, describes the remaining parameters chosen and moments matched. The model period is assumed to be a month. We set the probability of aging $\eta = \frac{1}{30 \times 12}$, implying an average of 30 years in the young state. The average time spent in the elderly state is assumed to be 20 years, therefore, $\rho = \frac{1}{20 \times 12}$. Lastly, we normalize the marginal product of the young, $p_y$, to be one, and choose the marginal product of the elderly, $p_o$, to match the earnings ratio in 2000 of $\frac{e_o}{e_y} = 1.23$.

The specified calibration yields labor’s shares for the young and elderly of $\frac{e_o}{p_y} = 0.62$ and $\frac{e_o}{p_o} = 0.55$. Consistent with the regression results, the model generates a lower share for the elderly workers, although the magnitude of the differences is smaller in the model. The endogenous lower share for elderly workers is driven by the lower endogenous match surplus due to a shorter expected duration, as well as the lower ratio of $\frac{z}{p_o}$.

Given the difference in the labor’s shares of the young and elderly, we now use the model to perform three counterfactuals to determine how much of the fall in the aggregate labor’s share can be accounted for by the model. The first counterfactual performed is to exogenously increasing the share of elderly workers, $N_o$, from the labor force share in 2000 to match the same share in 2013.\(^8\) The second counterfactual maintains the share of workers, but increases the product of the elderly, $p_o$, to match the relative earnings per worker in 2013. The final counterfactual combines both the change in the labor force and marginal product. Table 6 shows the results of the three counterfactuals.

The combined effect of changes in labor force and relative earnings generate a decline in the model aggregate labor’s share of 1.22%, which is one-third the predicted fall due to demographic changes (−3.62%) predicted in the accounting results. We see that most of the fall is attributed to the changes in the labor force; the fall in the model labor’s share is 0.84 when changing only the labor force, and the fall is 0.24 when changing only the earnings per worker. However, it is important to note that the combined effect

\(^8\)Since we have a 100% labor force participation rate, we use the total number of each age group to match changes in the labor forces.
is not simply additive, suggesting that there may be important interaction between the labor force and earnings per worker. Lastly, changes in earnings per worker for the elderly have almost no impact on the endogenous employment shares, thus the shift in elderly’s earning share is only through earnings per worker. However, changes in the labor force have significant changes in the employment share of the elderly which is driving the increase in the earnings share of the elderly. Thus, we see that the simple life-cycle search model described above is able to generate the lower labor’s share of elderly consistent with the accounting analysis, and is able to account for one-third the predicted fall in aggregate labor’s share due to demographic changes.

5.3 The Postel-Vinay & Robin Model

To Be Written

6 Conclusion

We have given an account of demographic changes in the United States over the last fifty years and how these changes have manifested in relative earnings. Using a very simple extension of production theory (which nests the neoclassical growth model), we have shown that the earnings shares of appropriately chosen groups should account for the aggregate labor’s share. The exact relationship between aggregate labor’s share and group-specific earnings shares takes a non-linear form and the theory dictates the appropriate weights for averaging.

Empirically, we document that the share of earnings going to older workers (older than 50) has risen dramatically in recent years, precisely when the decline in labor’s share has accelerated. Some of the rise in elderly earnings shares has been due to increased labor force attachment of baby boomers, but an increase in average earnings in this group has also been important. In fact, for the 60+ group, most of the rise in earnings share has been due to an increase in their relative earnings-per-worker.

Finally, we have proposed a theoretical connection between the elderly’s share of earnings and aggregate labor’s share that is deeper than enriching the production function. A search and matching model with life-cycle heterogeneity naturally generates a lower labor’s share for elderly workers. An increase in the share of elderly workers in the labor force leads to a unequivocal fall in labor’s share and a rise in earnings-per-worker for the elderly may also reduce the aggregate labor’s share.
References


7 Appendices

This appendix includes robustness and technical derivations where appropriate.

7.1 A2: Alternative Groupings

To be written

7.2 Alternative Methods of Handing Business Income

Have tried all to labor.

8 Figures and Tables

8.1 Figures
# 8.2 Empirical Tables

Table 1: Summary of Earnings Shares, 1962 to 2013

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Mean</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>ages 17-29</td>
<td>0.192</td>
<td>0.125</td>
<td>0.256</td>
</tr>
<tr>
<td>ages 30-39</td>
<td>0.263</td>
<td>0.214</td>
<td>0.317</td>
</tr>
<tr>
<td>ages 40-49</td>
<td>0.264</td>
<td>0.209</td>
<td>0.310</td>
</tr>
<tr>
<td>ages 50-59</td>
<td>0.199</td>
<td>0.151</td>
<td>0.266</td>
</tr>
<tr>
<td>ages 60+</td>
<td>0.082</td>
<td>0.058</td>
<td>0.136</td>
</tr>
</tbody>
</table>
Table 2: US regression results

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{1}{LS}$</td>
<td>$\frac{1}{LS}$</td>
<td>$\frac{1}{LS}$</td>
<td>$\frac{1}{LS}$</td>
<td>$\Delta \frac{1}{LS}$</td>
</tr>
<tr>
<td>$\sigma_{elders}$</td>
<td>0.790***</td>
<td>0.515***</td>
<td>0.567***</td>
<td>(0.221)</td>
<td>(0.114) (0.123)</td>
</tr>
<tr>
<td>t</td>
<td>0.00197***</td>
<td>0.00190***</td>
<td>(0.000326)</td>
<td>(0.000346)</td>
<td></td>
</tr>
<tr>
<td>$\frac{\alpha_e^{-1}}{\alpha_y}$</td>
<td>0.710***</td>
<td>(0.0493)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \sigma_{elders}$</td>
<td>0.819</td>
<td>(0.524)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>1.376***</td>
<td>1.403***</td>
<td>1.390***</td>
<td>(0.0641)</td>
<td>(0.0292) (0.0317)</td>
</tr>
<tr>
<td>Observations</td>
<td>52</td>
<td>52</td>
<td>52</td>
<td>52</td>
<td>51</td>
</tr>
<tr>
<td>Cragg-Donald F</td>
<td>705.1</td>
<td>43.84</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Table 3: Sectors

1. Natural resources and mining
2. Construction
3. Durable goods manufacturing
4. Non-durable goods manufacturing
5. Trade/Transportation and utilities
6. Information
7. Financial activities
8. Professional and business services
9. Education and health services
10. Leisure and hospitality
11. Other services

Table 4: US Sector Results: First-Differences

<table>
<thead>
<tr>
<th>Field, j</th>
<th>$\Delta\sigma_{elders,j}$</th>
<th>(p)-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4.487***</td>
<td>(1.011)</td>
</tr>
</tbody>
</table>

Observations: 264

Standard errors in parentheses

* \( p < 0.05 \), ** \( p < 0.01 \), *** \( p < 0.001 \)

8.3 Model Tables

Table 5: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0.99</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.0028</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.0042</td>
</tr>
<tr>
<td>$s$</td>
<td>0.026</td>
</tr>
<tr>
<td>$z$</td>
<td>0.955</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.052</td>
</tr>
<tr>
<td>$p_y$</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Table 6: Counterfactuals

<table>
<thead>
<tr>
<th>Moment</th>
<th>LF</th>
<th>Earnings</th>
<th>Both</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta LS$</td>
<td>-0.81%</td>
<td>-0.24%</td>
<td>-1.22%</td>
<td>-3.62%</td>
</tr>
<tr>
<td>$\Delta \frac{E_o}{E}$</td>
<td>10.9%</td>
<td>1.34%</td>
<td>12.5%</td>
<td>12.1%</td>
</tr>
<tr>
<td>$\Delta \frac{EMP_o}{EMP}$</td>
<td>10.1%</td>
<td>-0.02%</td>
<td>10.1%</td>
<td>9.8%</td>
</tr>
</tbody>
</table>

Data $\Delta LS$ refers to the predicted fall due to demographics.

Figure 1: Aggregate Labor Share
Figure 2: Age Distribution of Earnings Shares
Figure 3: Decomposition of Earnings Share - ages 50+
Figure 4: Decomposition of Earnings Share - ages 17-29
Figure 5: Decomposition of Earnings Share - ages 30-39
Figure 6: Decomposition of Earnings Share - ages 40-49
Figure 7: Decomposition of Earnings Share - ages 50-59
Figure 8: Decomposition of Earnings Share - ages 60+
Figure 9: Elder Population Share and Earnings Share
Figure 10: Elder Population Share and Earnings Share
Figure 11: Actual vs Predicted Labor Share
Figure 12: Actual vs Predicted Labor Share
Figure 13: Ratio of Earnings Wedges $\left( \frac{\hat{a}_j}{\hat{a}_1} \right)$ Estimates and 95% CI